REPORT RESUMES

ED 010 535 24

THE PROBLEMS OF UNDER ACHIEVEMENT AND LOW ACHIEVEMENT IN MATHEMATICS EDUCATION.

BY- SMALL, DWAIN E. AND OTHERS
FLORIDA UNIV., GAINESVILLE

REPORT NUMBER CRP-H-307 PUB DATE NOV 66
REPORT NUMBER BR-5-0694
CONTRACT OEC-6-10-145
EDRS PRICE MF-$0.18 HC-$3.52 88P.

DESCRIPTORS- *MATHEMATICS INSTRUCTION, *PROBLEM SOLVING, *UNDERACHIEVERS, *LOW ACHIEVERS, ABILITY IDENTIFICATION, ELEMENTARY SCHOOL STUDENTS, TEST CONSTRUCTION, GRADE 4, GRADE 5, GRADE 6, CASE STUDIES (EDUCATION), DIAGNOSTIC TESTS (EDUCATION), ACADEMIC ACHIEVEMENT, MATHEMATICAL CONCEPTS, *REMEDIAL MATHEMATICS, GAINESVILLE, FLORIDA

THE FACTORS RELATED TO UNDERACHIEVEMENT AND LOW ACHIEVEMENT IN MATHEMATICS LEARNING WERE STUDIED. USING AN INDIVIDUAL CLINICAL APPROACH, THE STUDY WAS CONDUCTED WITH TWO SAMPLE GROUPS, 12 UNDERACHIEVERS AND 11 LOW ACHIEVERS, IN GRADES 4-6. SCORES WERE OBTAINED ON CONCRETE, SEMICONCRETE, AND ABSTRACT REASONING LEVELS FOR THE MATHEMATICS EDUCATION AREAS OF LINEAR MEASUREMENT AND PLACE VALUE. ADDITIONAL EVALUATIVE DATA, BOTH OBJECTIVE AND SUBJECTIVE, WERE COLLECTED THROUGH INTENSIVE CASE STUDIES ON EACH STUDENT. NO CONSISTENT PATTERN WAS FOUND FOR EITHER GROUP IN ABILITIES TO FUNCTION ON THE THREE DIFFERENT LEVELS OF REASONING. SEVERAL DIFFERENCES, HOWEVER, WERE FOUND BETWEEN THE TWO GROUPS WHEN EVALUATIVE DATA ON THEIR HOME ENVIRONMENTS AND SOCIAL ADJUSTMENTS WERE COMPARED. PROJECT RECOMMENDATIONS WERE BASED ON THOSE DIFFERENCES--(1) LOW ACHIEVERS NEED TO BE EXPOSED TO COMPREHENSIVE COUNSELING AND REMEDIAL PROGRAMS BASED ON THEIR INDIVIDUAL LEVELS OF ABSTRACTIVE ABILITY, AND (2) UNDERACHIEVERS SHOULD PARTICIPATE IN SPECIAL PROGRAMS DESIGNED TO REDUCE ANXIETY TOWARD MATHEMATICS. (JH)

ED010535

U. S. DEPARTMENT OF HEALTH, EDUCATION AND WELFARE
Office of Education
This document has been reproduced exactly as received from the person or organization originating it. Points of view or opinions stated do not necessarily represent official Office of Education position or policy.

FINAL REPORT
Project No. H-307 (5-0694)
OE 6-10-145

THE PROBLEMS OF UNDER ACHIEVEMENT AND LOW ACHIEVEMENT IN MATHEMATICS EDUCATION

November 1966

U.S. DEPARTMENT OF
HEALTH, EDUCATION, AND WELFARE

Office of Education
Bureau of Research

THE PROBLEMS OF UNDER ACHIEVEMENT AND LOW ACHIEVEMENT IN MATHEMATICS EDUCATION

Project No. H-307
Contract No. OE 6-10-145

Dwain E. Small
Donald L. Avila
Boyd D. Holtan
Kenneth P. Kidd

November, 1966

The research reported herein was performed pursuant to a contract with the Office of Education, U. S. Department of Health, Education, and Welfare. Contractors undertaking such projects under Government sponsorship are encouraged to express freely their professional judgment in the conduct of the project. Points of view or opinions stated do not, therefore, necessarily represent official Office of Education position or policy.

University of Florida
Gainesville, Florida

CONTENTS

Page No.

Acknowledgements iii

Chapter I - Introduction 1

Chapter II - Methods 5

Chapter III - Results 12

Chapter IV - Discussion 29

Chapter V - Conclusions, Implications, and Recommendations 33

Chapter VI - Summary 35

References 38

Appendix A - Linear Measurement Test Concrete

Appendix B - Linear Measurement Test Semi-Concrete

Appendix C - Linear Measurement Test Abstract

Appendix D - Place Value Test Concrete

Appendix E - Place Value Test Semi-Concrete

Appendix F - Place Value Test Abstract

Appendix G - Behavior Observation Guide

Appendix H - How I See Myself and Family Essay Form

ACKNOWLEDGEMENTS

This project would not have been possible without the cooperation and support of many people. The staff is deeply indebted to the P. K. Yonge Laboratory School for its cooperation in the pilot phase of the testing program and to the Alachua County Public Schools for the use of the students' time and school facilities.

The staff owes special thanks to Mr. W. L. Kilpatrick and Mr. M. H. Parramore of the Alachua County School System and to the teachers who were so patient during the testing periods.

To the graduate assistants, Mrs. Lonita McGill and Mr. Joseph Moretta, the staff is forever indebted. Their persistence in the gathering of data was indispensable to the project.

The secretaries who worked at various stages of the project, Mrs. Marcia Buchanan, Mrs. Margaret Faloona, and Mrs. Christina Sollosso, were helpful and efficient. They deserve our appreciation.

CHAPTER I

INTRODUCTION

PROBLEM

Many of the economic and social problems which are constantly plaguing the advancement of our society can be attributed partially to the inability of educators to solve the problems of low achievement and under achievement in our schools. On November 14, 1961, the Wall Street Journal carried a front page article entitled "School Drop-Outs" which observed that "youths not finishing studies create growing economic problems."

In discussing reasons for drop-outs, Mr. Downey, Director of Vocational Guidance for the Boston Schools, was quoted, "In most cases they (drop-outs) have had a difficult time with their school work, have failed one or more times..."

According to the National Committee for Children and Youth, the majority of drop-outs are labeled "slow learners." However, 20 - 25 per cent of them are said by this committee to "possess superior intelligence."

The economic concomitants of automation and a large uneducated labor supply constitute a serious national problem with social, economic and political implications. In the December 1961 issue of Education U.S.A., it was stated that "The problem of the under achiever is deemed to be specially important because of the decline in opportunities for unskilled workers and the rise (46 per cent in eight years prior to 1958) in occupations which may be classed as professional. It is estimated that by 1975 the country will need twice as many scientists and engineers as it had in 1958."

In Guidance for the Under Achiever with Superior Ability, an 85-page report published by the U. S. Office of Education, the following findings are reported:

1. Between 20 per cent and 30 per cent of able students in the nation's high schools and colleges do not achieve the full measure of success their abilities warrant.

2. No one incisive factor identifies all under achievers.

3. Early training in the home and in the school is thought to be an important factor affecting matters other than mental

health which determine success or failure in school.

4. Usually the first thing a teacher does when confronted with under achievement in a student is to recommend some form of special instruction such as tutoring or remedial reading. These steps are often effective in some degree, but they do not usually reverse the patterns of thinking and behavior which have been part of the student's "life style" for many years.

5. At the elementary school level, the report recommends that the school staff should recognize that there are readiness levels for all types of learning, and wide differences within each individual, as well as between individuals.

RELATED RESEARCH

A survey of the literature reveals a rather familiar pattern of data related to drop-outs and under achievement.

Cook (2) differentiated between reasons given by students for dropping out, and those given for the same cases by counselors: failure and retardation, 35 per cent; home circumstances, 28 per cent; conflicts with teacher, 7 per cent; and feelings of rejection, 9 per cent. As early as 1922 Counts (3) showed a positive relation between low economic status and withdrawal. He also found that national origin and race were related to survival in school. His findings were corroborated in 1942 by Karpinos and Sommers (10), who reported attendance rates of 67.2 per cent for boys, aged 15 to 19, from homes with incomes under $1000 per year, and 95.9 per cent for boys from families with incomes above $3,000. The corresponding percentages for girls were 71.6 and 93.7.

In summarizing the cause of early school leaving Cummings and others (4) state that a complexity of influences are brought to bear upon the potential drop-out. Among these are school, home and community experiences.

Dillon (6) concludes that factors which reflect dissatisfaction or difficulty in school are of major importance. Penty (12) found that more than three times as many poor readers as good readers drop out of school.

Cook (2) reported that failure was the major cause for about one-third of the drop-outs. Dillon (6) found a closer relationship: 52 per cent of the 1300 early school leavers in his study had failed in one or more grades while in elementary school, and 74 per cent had failed in one or more high school subjects. Fourteen per cent had failed in one subject, 13 per cent in two, 17 per cent in three, and 30 per cent in four or more subjects. DeLaney (5) found only 19 per cent of 7177 drop-outs in Chicago high schools not failing in many or all subjects.

The chief causes of failure found by Carrothers (1) were excessive teacher load in or out of school, lack of student interest, lack of understanding of the student by teacher, inability of the student to do the work expected, parental unconcern, community misunderstanding of educational needs, the school's inability to measure student growth and to report progress to studentand community, "spoon feeding" at home and school, and inflexibility of curriculum and requirements for student and teacher. Similar reasons had been reported earlier by Farnsworth and Casper (9), particularly the lack of student interest, which was listed by 77 per cent of the students in their Utah study. Lafferty (11) found that 24 per cent of the learning difficulties were due to lack of student ability and that 76 per cent were due to conditions for which the school was essentially responsible.

Several studies have considered the student of ability who fails. DeLaney (5) reported that 46 per cent of the 7177 drop-outs in his study were of average or better-than-average mental ability. Durrell (7) states that 25 per cent of the children who make slow progress are of normal or superior intelligence, and points to such causes as mental malfunctioning, unfavorable emotional and personality factors, and poor physical conditions. Edelston (8), in his study of British 12 and 13-year-olds of high intelligence who were failing, distinguished the following categories clinically: children with disturbing home conditions; reaction character formations; infantile neuroses; constitutional defects of character, including grave defects in emotional maturation; and psychopathic states and early psychoses. He makes clear that limitations to learning imposed by such emotional states are more rigid than commonly believed.

Strom (14) concluded that until potential drop-outs are individually helped to succeed, schools collectively will fail. Whaley (17) suggested that the most effective single drop-out preventive device is an understanding, sympathetic teacher who can help a child realize the best that is in him.

For the most part these and other studies are descriptive and seek to classify the conditions surrounding under achievement. There is little evidence that such studies have resulted in new or extensive use of methods or materials for academic remediation. Such school surveys usually result in recommendations of: (1) referrals to counseling, (2) conferences with parents, or (3) placement of students in remedial classes composed of numbers of "similar" learning cases.

PURPOSE OF STUDY

It was the purpose of this pilot study to explore factors related to low achievement and under achievement in mathematics education and to determine if there are individual levels of abilities in abstractive thought with respect to mathematics concepts. It was hoped that this exploratory phase would lead to hypotheses which could be tested through extensive study.

OBJECTIVES OF STUDY

This pilot study sought new approaches to remediation in mathematics for the low achievers and under achievers. The major objectives of the study were to:

1. Formulate a flow chart for the development of concepts in two mathematical areas at grades 4 - 6.

2. Select three levels within the concrete-abstract continuum for each stage of the development of the concept.

3. Test the abilities of a group of low achievers and under achievers in mathematics to function with a concept at each of three selected levels of the concrete-abstract continuum.

4. Make an intensive analysis of each student using the case study technique.

HYPOTHESES

No hypotheses were formulated for testing in this study since the purpose of the study was to locate problems which would lead to hypotheses for further investigation.

CHAPTER II

METHOD

A. General Methods

Beginning in September 1965, the mathematics staff planned the sequence of mathematical ideas to be tested. Items were constructed and pilot tests were administered at the P. K. Yonge Laboratory School on the University of Florida campus to a selected group of 4th, 5th and 6th grade students. Revision of the test items was completed by December 1965 and plans were formulated for contacts with the Alachua County, Florida, School System for receiving permission to begin the research at one of the elementary schools in Gainesville, Florida.

A statement of the research objectives along with an application for cooperation and permission to conduct the research was submitted to the Alachua County Board of Public Instruction. The application was approved and J. J. Finley Elementary School was recommended by the Alachua County Assistant Superintendent.

The principal of J. J. Finley School was contacted confirming his cooperation and informing him that a visit to the school by the staff would soon be made to work out a satisfactory time schedule for the research.

The initial personal contact with J. J. Finley School and staff was made during the third week of January 1966. At that time, the mathematics education team, the project psychologist, and two graduate research assistants met with the principal in his office to plan the details of the project. The principal agreed to permit the staff complete freedom regarding sample selection, access to school records, scheduling pupil interviews and tests, and assured the research staff that all teachers were willing to cooperate by excusing students to participate and by making available any classroom records which were needed for the purposes of research data.

The subjects for the sample were selected from the fourth, fifth and sixth grade classes, as these particular years are felt by the project staff to be the most indicative of a child's trend in performance and possible need for remediation.

The research proposed to study two groups; one group of ten subjects defined as under achievers in mathematics and a second group of ten students defined as low achievers.

The initial selection of each group was made on the basis of the students' scores on the Stanford Achievement Test which had been administered to the students during the previous school year (September 1965). All records and test scores were made available to the research staff.

The following criteria were used to determine the selection of subjects. A subject was considered an under achiever in mathematics if his non-mathematics scores were equal to or above his present grade placement, but his average percentile score in mathematics computation and concepts was two or more deciles below his non-mathematics percentile average. (The Arithmetic Application section of the Stanford Test was not included since the scores on this section seemed to be highly inconsistent with the other two mathematics portions that were used in the study.)

A subject was considered a low achiever if his average percentile scores on all sections of the Stanford Test were at least two deciles below his present grade placement level.

A final stipulation of selection was that all of the subjects be at least average in measured intelligence. I.Q. scores were obtained from the subjects Cumulative Records, and "average" intelligence was defined in terms of Wechsler's (1958, p.42) table of I.Q. distributions.

The research assistants initially selected 41 students, (18 mathematics under achievers and 23 low achievers). From this initial sample the project staff selected those students whose achievement scores provided the best approximations to the selection criteria.

The final sample selected by the staff for the research consisted of 25 students (13 in the under achiever group and 12 in the low achiever group). As was stated earlier, two groups of ten students each were desired, but a larger sample was chosen to insure a measure of safety in view of the fact that a student may have declined to participate, become ill, or leave school for some reason during the period of the research. Two students chosen for the final sample did withdraw from the school, leaving the final sample size of 23 students (12 mathematics under achievers and 11 low achievers).

The list of students selected by the staff was submitted to the principal who contacted the parents of each student, briefly outlining the project objectives and asking permission to include their child in the study.

A brief letter was sent to each student's parents along with a form to be checked by the parent, either approving or disapproving of his child's participation in the study. This form also requested the parent to check the time which would be most convenient for a home interview. The home interview was to be a required facet of the study and is described in the Psychological Evaluation Procedures.

All but two permission forms were returned by the parents within two weeks, and no forms were returned with negative replies. Parent cooperation was most commendable.

A research assistant contacted each subject's parents and scheduled the home interview appointments. It was predicted that the home interviews each would require more than one hour of time. Hence, the home visits were scheduled to provide a maximum of convenience to the participating parents.

B. Mathematics Methods

Two areas of mathematics education were selected for study. They were linear measurement and place value. Linear measurement was selected because its development in school programs is often incidental and hence, it was felt that the student's concepts in this area might be weaker.

Place value is basic to the development of skill in the four fundamental operations. It was felt that testing in this area would be a necessary preliminary to later testing of the operations when the study is continued.

The staff studied the area of linear measurement and identified the following sequence of ideas:

1. Comparison of length
    a. vocabulary
    b. non-uniform units
    c. conservation
    d. transitivity

2. Concepts of linear measure
    a. arbitrary units
    b. comparison of units
    c. standard units
    d. commensurability
    e. approximation
    f. precision

3. Construction and Use of Ruler
    a. zero as the origin
    b. origin adjustment
    c. uniform units
    d. ordinal idea of units on a ruler

The place value test was planned by the staff utilizing the following sequence of ideas:

1. Counting

    a. counting by ones, two's, fives and tens
    b. counting by groups of tens and ones

2. Grouping

    a. efficiency created by grouping
    b. effects on number by adding groups to the number
    c. digit to be changed when additional groups are given

3. Number - Numeral

    a. forming numeral when numbers of things are given
    b. illustrating using numbers of things when numeral is given
    c. reading numerals

Each concept to be tested was carefully identified and items formulated on three levels of the concrete - abstract continuum; (a) concrete (b) semi-concrete or pictorial (c) abstract or verbal-symbolic. As operating definitions for this study, the following were formulated:

(a) An item was concrete when the material was the physical model itself and could be manipulated by the subject.

(b) An item was semi-concrete when the material was presented as a model in the pictorial level, such as, a photograph or a drawing.

(c) An item was abstract when the material was presented verbally without any physical model to assist the subject.

A test on linear measurement was constructed for each of these three levels of the concrete-abstract continuum. The same three levels were used to construct three tests on place value. (See Appendix (A-F)for samples of the tests.

The tests were administered orally by the examiner and the student was asked to give oral answers which were recorded by the examiner. Each test required between 15-30 minutes of the student's time.

The student's reactions to an item as well as oral responses were recorded by the examiner. When a test was completed, the examiner recorded his impressions of the student's performance and scored each item of the test. A scoring system of 1 to 4 points for each item was used as follows:

4 - Student definitely has the concept.
3 - Student probably has the concept.
2 - Student probably does not have the concept.
1 - Student definitely does not have the concept.

The abstract test for linear measurement was administered first to all students. This required four days of scheduled testing. When all of the students had completed the abstract test of measurement, the cycle of testing was repeated using the abstract test of place value.

The next cycle of testing was the semi-concrete tests of measurement and place value. These followed by one week the abstract tests. The semi-concrete linear measurement test was administered utilizing the overhead projector to present the models. This method permitted the staff to use real devices and translate them to the dimension of pictures.

To present the semi-concrete test on place value, photographs were made of bundles of sticks. These photographs were presented to the student as each question was asked by the examiner. It should be noted at this point that within the concrete-abstract continuum for a concept there are a multiple number of levels. The semi-concrete level is a subset of the continuum and is a continuum itself. Hence, of all the possible ways of forming a pictorial model for the concept, it was judged by the staff that the photograph would be the truest model.

When the semi-concrete cycle of testing was completed, the concrete tests were administered. Kits of materials were developed for each item so that the three examiners would present the same material for each item when testing. Because of the need to manipulate more materials and keep the kits in proper order, the time for administering the tests increased. It is possible that the time increase is a reason for this type of test not being favored in a comprehensive diagnostic testing program.

C. Psychological Methods

The procedure followed for collecting the psychological and sociological data on the subjects of this study was a modified case study technique which is employed in order to establish a data pool. This technique is described by Shontz (13).

> "It sometimes happens that case material is collected for purposes that cannot be particularized in advance. Investigators with an interest in special phenomena may exert efforts to collect case study information for later analysis, but they may not have in mind the exact approach that will be taken to their data once the data become available. They simply accumulate a type of information that their professional judgment suggests will be useful and valuable to themselves or others when a sufficient amount of it has been obtained." (p.73)

The data pool for the present study was established on the basis of both objective and subjective information gathered from the following sources.

Objective Data

1. Cummulative Records: This source of data was simply the cummulative folders maintained on each student by the school.

2. California Personality Inventory (CPI): This is a standardized personality inventory. It was administered in a group testing situation at the beginning of the study.

3. Wechsler Intelligence Scale for Children (WISC): This standardized test of intelligence was administered to each child individually by the staff psychologist.

4. Teachers Rating Scale of Pupil Adjustment: This scale was distributed to each teacher individually by one of the project assistants, and later retrieved by the same person after the teachers had completed the forms. Like the CPI, this scale is a standardized measure.

Subjective Data

1. Psychological Observation: The project psychologist made subjective observations of each child, after which he proceeded to fill out a Behavior Observation Guide (BOG - See Appendix G).

2. How I See Myself and Family: The project psychologist developed this method of attempting to ascertain, to some degree, the personal and familial adjustment of the subjects. Each subject was asked to respond to a statement (Appendix H) which dealt with the way he felt about himself and family. This instrument was administered after the administration of the WISC.

3. Teachers Interviews: A graduate assistant interviewed each teacher participating in the study. Information was gathered from each teacher regarding her students, such as, academic performance and general behavior.

4. Parent Interviews: A graduate assistant went to the home of each subject and personally interviewed at least one adult in each home. The adult interviewed in most cases was one or both of the subject's parents. Information was collected, such as, general socio-economic status, parental age, education and occupation, and family harmony.

CHAPTER III

RESULTS

A. Mathematics

An objective of this study was to test the abilities of a group of low achievers and under achievers in mathematics to function with a concept at each of three selected levels of the concrete-abstract continuum. For the purposes of research reporting, group data was used. However, if individual programs of remediation were to be developed, the performance of each student would be the primary consideration.

From an examination of Table 1 and Table 2, it is apparent that there are variations in the abilities of the students of this study to function with a concept at each of the three selected levels of the continuum.

Table 1 - MEANS OF LOW ACHIEVERS' SCORES ON DIFFERENT ABSTRACT LEVELS OF THE LINEAR MEASUREMENT TEST

Student	Abstract Levels		
	Abstract	Semi-Concrete	Concrete
1	1.4	2.6	2.7
2	1.9	2.9	3.2
3	2.3	3.6	3.5
4	2.9	3.7	3.5
5	2.5	3.4	3.8
6	1.6	2.3	2.7
7	2.1	2.6	3.0
8	2.4	3.2	3.7
9	3.1	3.6	3.6
10	3.1	3.6	3.8
11	3.5	3.2	3.9

Recall that from Chapter II, the following scoring system was used:

4 - Student definitely has the concept
3 - Student probably has the concept
2 - Student probably does not have the concept
1 - Student definitely does not have the concept

Hence, a mean score for a student on any test of between 2.8 and 3.2 was interpreted as a case where the student probably could function effectively at that level of the continuum on that conceptual area.

Table 1 shows that four (4) of the low achievers probably could function at the abstract level of linear measurement, eight (8) could function at the semi-concrete level, and all of them could function effectively at the concrete level.

Table 2 shows that six (6) of the under achievers probably could function at the abstract level of linear measurement, eleven (11) would function at the semi-concrete level, and all of them could function at the concrete level.

Table 2 - MEANS OF UNDER ACHIEVERS' SCORES ON DIFFERENT ABSTRACT LEVELS OF THE LINEAR MEASUREMENT TEST

	Abstract Levels		
Student	Abstract	Semi-Concrete	Concrete
1	2.1	2.9	3.6
2	1.8	2.6	3.4
3	2.1	3.3	3.6
4	3.2	3.8	3.8
5	2.3	2.9	3.4
6	2.6	3.1	3.6
7	3.2	3.2	3.7
8	2.9	3.2	3.6
9	3.1	3.1	3.6
10	3.4	3.5	3.9
11	3.1	3.0	3.8
12	2.6	3.5	3.8

Table 3 indicates the means of scores for each group on the different levels of the linear measurement test by actual grade of the students. Observe that the sixth grade students probably could function on the abstract level and the fifth grade under achievers could also. However, the fifth grade low achievers and all of the fourth grade students probably could not function on the abstract level.

Table 3 - MEANS OF SCORES BY ACTUAL GRADE FOR BOTH GROUPS ON LINEAR MEASUREMENT

Actual Grade	Achievement Group	Abstract Levels: Abstract	Semi-Concrete	Concrete
4	Low	2.1	3.2	3.2
	Under	2.3	3.1	3.6
5	Low	2.1	2.9	3.3
	Under	3.0	3.2	3.6
6	Low	2.8	3.2	3.6
	Under	3.0	3.3	3.8
Total Group	Low	2.4	3.2	3.4
	Under	2.7	3.2	3.7

Table 4 indicates the means of scores for each group on the different abstract levels of the linear measurement test by the achievement test grade in mathematics of the students. Note that if the students had a fifth or sixth achievement test grade in mathematics, or fourth grade and under achieving, they probably could function on the abstract level of linear measurement.

Table 4 - MEANS OF SCORES BY ACHIEVEMENT TEST GRADE IN MATHEMATICS FOR BOTH GROUPS ON LINEAR MEASUREMENT

Achievement Test Grade	Achievement Group	Abstract Levels		
		Abstract	Semi-Concrete	Concrete
2	Low	---	---	---
	Under	2.2	2.9	3.5
3	Low	1.9	3.0	3.1
	Under	2.0	3.0	3.5
4	Low	2.4	3.1	3.3
	Under	2.9	3.5	3.7
5	Low	2.9	3.4	3.8
	Under	3.0	3.3	3.8
6	Low	---	---	---
	Under	3.2	3.2	3.7

If the results of the place value tests are analyzed in a like manner, we can see a trend similar to that for linear measurement. Table 5 and Table 6 show the means of the scores for each student on the place value tests. From Table 5, it can be noted that four(4) of the low achievers probably could function at the abstract level and all at the semi-concrete level of the place value concepts tested.

From Table 6, it can be noted that six (6) of the under achievers probably could function at the abstract level and all at the semi-concrete level of the place value concepts tested. This place value result differed from the linear measurement result, since all could probably function on the semi-concrete level of place value, while four (4) probably could not function on the semi-concrete level of linear measurement.

Table 5. MEANS OF LOW ACHIEVERS' SCORES ON DIFFERENT ABSTRACT LEVELS OF THE PLACE VALUE TEST

Student	Abstract Levels Abstract	 Semi-Concrete	 Concrete
1	1.4	3.4	3.5
2	1.6	3.3	3.6
3	2.3	3.8	3.8
4	3.6	3.8	3.8
5	2.6	3.9	3.95
6	1.2	3.1	3.4
7	2.0	3.0	3.7
8	3.6	3.8	3.8
9	2.5	3.9	3.9
10	2.9	4.0	4.0
11	3.95	4.0	3.95

Table 6. MEANS OF UNDER ACHIEVERS' SCORES
ON DIFFERENT ABSTRACT LEVELS
OF THE PLACE VALUE TEST

	Abstract Levels		
Student	Abstract	Semi-Concrete	Concrete
1	2.0	3.4	3.8
2	2.1	3.3	3.7
3	1.4	3.3	3.4
4	3.3	3.9	3.8
5	2.2	3.7	3.9
6	1.9	3.9	3.95
7	3.1	4.0	3.95
8	2.4	3.9	3.9
9	3.1	3.9	4.0
10	4.0	3.8	4.0
11	3.6	3.8	3.8
12	3.95	4.0	4.0

Table 7 and Table 8 present an analysis of place value by actual grade and achievement test grade in mathematics. If a student of this study is in the sixth grade, he probably can function at the abstract level of place value. However, if the achievement test grade is used, a fifth or sixth test grade would indicate that the student probably could function at the abstract level of place value.

The value of this type of analysis can be realized if we observe the performance of individual students. Observe from Table 1 and Table 5 that students nos. 1 and 6 probably could function only on the concrete level of linear measurement. However, these two students probably could function on the semi-concrete level of place value.

Student no. 2 of the under achiever group is the only under achiever who probably could not function at the semi-concrete level of linear measurement. However, this student probably could function at the semi-concrete level of place value.

Table 7 - MEANS OF SCORES BY ACTUAL GRADE FOR BOTH GROUPS ON PLACE VALUE

Grade	Achievement Group	Abstract Levels Abstract	Semi-Concrete	Concrete
4	Low	2.2	3.6	3.7
	Under	2.2	3.5	3.7
5	Low	1.9	3.5	3.7
	Under	2.6	3.9	3.95
6	Low	3.0	3.7	3.9
	Under	3.8	3.9	3.9
Total Group	Low	2.5	3.6	3.8
	Under	2.8	3.7	3.9

Table 8 - MEANS OF SCORES BY ACHIEVEMENT TEST GRADE IN MATHEMATICS FOR BOTH GROUPS ON PLACE VALUE

Achievement Grade Level	Achievement Group	Abstract Levels		
		Abstract	Semi-Concrete	Concrete
2	Low	---	---	---
	Under	2.1	3.6	3.9
3	Low	1.8	3.5	3.6
	Under	1.8	3.3	3.6
4	Low	2.4	3.5	3.7
	Under	2.6	3.9	3.9
5	Low	3.2	3.9	3.9
	Under	3.5	3.9	3.9
6	Low	---	---	---
	Under	3.1	3.95	3.98

B. Psychological

After the data pool was established the decision was made to analyze the data by comparing the low and under achievers on each of the individual sources of data. These comparisons yielded the following information:

Objective Data

1. Cumulative Records: These records did not prove to be a satisfactory source of data. In each case the information included in these records was sketchy and incomplete and as it happened, any useful information that was contained in them was eventually obtained from one of the other sources.

2. California Personality Inventory: Table 9 presents the mean scores for the low and under achievers on all sub-scales of the CPI, as well as the personal, social and total adjustment scores.

Table 9. *MEANS FOR LOW AND UNDER ACHIEVERS ON ALL SCALES OF THE CALIFORNIA PERSONALITY INVENTORY

	Low Achievers	Under Achievers
Self Reliance	44.54	48.33
Personal Worth	54.54	59.00
Personal Freedom	51.36	39.16
Belongingness	51.81	47.66
Withdrawal	58.18	54.16
Nervous Symptoms	48.63	58.33
Total Personal	41.81	44.16
Social Standards	55.00	61.66
Social Skills	38.63	50.58
Anti-Social Tendencies	30.09	24.00
Family Relations	42.00	43.41
Occupational Relations	40.90	55.00
Community Relations	52.72	50.41
Total Social	38.18	44.58
Total Adjustment	39.09	45.83

* None of the differences were significant.

An inspection of Table 9 reveals that the personal, social and total adjustment of both groups are below the norm average of the CPI standardization group. The major problems of the low achievers appear to be a lack of social skills, rather strong anti-social tendencies, and a high degree of nervous symptoms. Their highest adjustment scores, on the other hand, were obtained in the areas of a sense of personal worth, low withdrawal tendencies, and high social standards.

The major problems of the under achievers, as indicated by the CPI, were a lack of personal freedom and very strong anti-social tendencies. The highest adjustment scores were obtained by the under achievers in the areas of a sense of personal worth, low withdrawal symptoms, high social standards and good occupational relations.

When t-test of the means between groups were ran, none of the differences were significant.

It should be kept in mind that the highest adjustments scores obtained by the members of these two groups are only high in comparison to their other scores, but still do not suggest outstanding adjustment when compared to the CPI standardizing sample.

3. Wechsler Intelligence Scale for Children (WISC): Because of the limited time alloted for administration of the WISC, only four of the performance scales were administered to the subjects. The performance and total IQ's were obtained by Wechsler's (16) prorating techniques.

Table 10 presents the scale score means for the low and under achievers on each of the WISC sub-scale tests administered.

Table 10 - SCALE SCORE MEANS FOR LOW AND UNDER ACHIEVERS ON ALL SUBSCALES OF THE WISC

	Low Achievers	Under Achievers	P
Information	11.36	15.41	.01
Comprehension	11.63	14.50	.01
Arithmetic	11.18	11.25	NS
Similarities	11.54	14.50	.01
Vocabulary	11.63	14.83	.01
Picture Completion	11.70	12.72	NS
Picture Arrangement	10.80	12.45	NS
Object Assembly	9.50	12.54	.05
Coding	12.10	14.36	.05

An inspection of Table 10 reveals a significant difference between the means of the low and under achievers on information, comprehension, similarities, vocabulary, object assembly, and coding, with no differences noted with regard to arithmetic, picture completion, and picture arrangement.

Table 11 presents the WISC verbal, performance and total IQ's for the low achievers and Table 12 presents the same data for the under achievers.

Table 11 - VERBAL, PERFORMANCE AND TOTAL IQ'S FOR LOW ACHIEVERS

S		V(N=11)	P(N=10)	T(n=10)
1		92	97	94
2		123	127	127
3		115	*	
4		100	108	104
5		116	127	123
6		92	78	84
7		97	83	90
8		119	113	117
9		108	97	103
10		110	111	112
11		129	133	134
	M =	109.18	107.40	108.80

*Due to extraneous circumstances the performance test was not administered to subject number 3.

Table 12 - VERBAL, PERFORMANCE AND TOTAL IQ'S FOR UNDER ACHIEVERS

S	V(N=12)	P(N=11)	T(N=11)
1	134	121	131
2	103	115	109
3	110	114	113
4	140	132	140
5	128	*	
6	108	100	104
7	143	127	138
8	116	114	117
9	134	113	126
10	143	120	135
11	115	143	131
M =	125.91	121.00	125.63

***Due to extraneous circumstances the performance test was not administered to subject number 5.**

Table 13 presents the mean verbal, performance and total IQ's for low and under achievers along with the results of tests of the significance of the difference between these means. An examination of Table 13 reveals significant group differences between verbal and total IQ's, but not between performance IQ.

Table 13. VERBAL, PERFORMANCE AND TOTAL IQ MEANS FOR LOW AND UNDER ACHIEVERS

	Low Achievers	Under Achievers	P
V	109.18	125.91	.01
P	107.40	121.00	NS
T	108.80	125.63	.01

4. Teachers' Rating Scale of Pupil Adjustment: Table 14 presents the teachers' mean ratings of the subjects on the Rating Scale of Pupil Adjustment. Each subject was rated on each factor (emotional adjustment, social maturity, happiness, etc.) on a five-point scale from 1 to 5 for a total possible rating of 45 points. The higher the score, the more positive the teacher rating. It may be seen from an inspection of Table 14 that the teachers consistently rated the under achievers higher than the low achievers with the two exceptions of aggressiveness and irritability. The mean total ratings of the teachers of the low and under achievers were 33.18 and 36.08, respectively.

Subjective Data

1. Psychological Observation: The project psychologist used the occasion of the administration of the WISC and How I See Myself and Family Essay (HISMF) to make a subjective evaluation of the subjects behavior.

Table 14. MEAN TEACHER RATINGS OF LOW AND UNDER ACHIEVERS

	Low Achievers	Under Achievers
Emotional Adjustment	3.72	3.91
Social Maturity	3.18	3.66
Happy	4.18	4.41
Aggressive	4.45	4.41
Emotional Security	3.90	4.00
Motor Control	3.81	4.00
Irritability	4.09	3.91
School Adjustment	2.54	3.91
School Conduct	2.27	3.83

Following each administration of these two instruments the psychologist proceeded to fill out a Behavior Observation Guide (BOG) (Appendix G) which was modified from the guide suggested by Watson (15). Besides checking the appropriate descriptive categories listed on the BOG, the psychologist made pertinent notes on each subject. Later, the information from these observations was analyzed in terms of comparing the low and under achievers.

The subjects' responses to the HISMF essay-type question proved to be a disappointing source of data. The subjects' written statements were short, superficial and factual. It was hoped that this device would reveal to some degree the nature of the subjects' personal adjustment and family relationships. Unfortunately, such was not the case. Consequently, the HISMF data was not considered at length.

A comparison of low achievers and under achievers from the data obtained from the psychologist's observations was made by taking the following steps: (1) The BOG data including the psychologist's notations for each subject were independently and carefully considered in an attempt to discover any outstanding behavioral characteristics or patterns that might have been noted in regards to the subjects at the time of the observation.

(2) on the bases of these considerations an interpretive paragraph was written which constituted a description of each subject; (3) from an analysis of these paragraphs a brief statement was constructed which, in the opinion of the psychologist, delineated the most outstanding characteristic of a subject as observed from his behavior.

Table 15 presents these brief statements for each subject. It may be seen from an examination of Table 15 that, even though the psychologist did not know which group a given subject was in when he constructed these statements, when the statements are compared there is a fairly consistent differentiation that can be made between the two groups in terms of the amount of anxiety manifested. Furthermore, it is consistent with the results obtained from the CPI, that the psychologist observed some form of adjustment difficulty in all but five of the subjects.

Table 15. PSYCHOLOGIST'S CHARACTERIZATION OF EACH SUBJECT

Low Achievers	Under Achievers
Appropriate Behavior	Very Anxious
Appropriate Behavior	Very Anxious
Slightly Anxious	Withdrawn
Feelings of Inadequacy	Appropriate Behavior
Feelings of Inadequacy	Very Anxious
Feelings of Inadequacy	Very Anxious
Rigid	Appropriate Behavior
Very Disorganized	Very Anxious
Lethargic	Withdrawn
Tense, Withdrawn	Feelings of Inadequacy
	Very Anxious

2. Teacher Interviews: The interviews with the teachers yielded information very much like that gained from the teachers' ratings. The teachers consistently characterized the low achievers as being poor students, as having more personal problems and as being less well socially adjusted, while characterizing the underachievers in the opposite manner.

3. Parent Interviews: The parent interview results revealed the following combination of subjective and objective data:

a. Siblings: The low Achievers had an average of 2.1 siblings living in the home and the under achievers had an average of 1.8. The range of siblings living with the low achievers was 1 to 4 and with the under achievers 0 to 2. The modal number of siblings for the underachievers was 2 and for the low achievers 3.

b. Parents'age: The mean age of the low achievers' fathers was 41.7, for their mothers 39.4 and for both parents 40.5. The age range of the low achievers' parents was 32 to 55.

The mean age for the under achievers' fathers' was 40.4, for their mothers 38.2, and for both parents 39.0. The age range of the under achievers' parents was 30 to 57.

c. Occupations and Interview Evaluation of Homes: The standard of living and occupational status of all of these families, as judged by the research assistant making the home visits, was above average, and judged by the same person in terms of socio-economic factors to be in the middle and lower upper class. Almost all of the parents' occupations in both groups could be classified as professional, including physicians, university professors, attorneys and business owners in each group.

d. Attitude of Parents: The project psychologist and research assistant attempted to evaluate the attitude of the parents towards their children from the type of statements the parents made during the interview. Although there were a few cases where the parents' statements clearly left one with the impression that they felt strongly positive about their children, the general conclusion drawn from this evaluation was that, by and large, most of the parents interviewed were expressing a marked ambivalence in their attitudes towards their children. Several parents stated that they found it very difficult to communicate with their offsprings. The comment was often made by the parents that they had little in common with their children, and quite often the children spent a good deal of time with someone other than the parent.

Review of the interview transcripts also revealed a rather large number of "critical events" having occurred in the life of the subjects in both the low and under achieving groups. The following are offered as examples of this:

Case V: Parents divorced. Children live with father because mother has been declared unfit. She has record of several arrests, and has recently just gotten out of jail.

Case W: Child's father is psychotic patient.

Case X: Subject's brother is patient in mental institution. Father does not communicate with nor engage in activities with other children.

Case Y: Mother stated she has never liked children very much, including, until quite recently, her own. She feels that she is beginning to like her own a little more now, and believes that she will start being a better mother than she has been in the past.

Case Z: Parents have always treated children rather badly, and at present employ severe corporal punishment upon little provocation.

The only factor gleaned from the interviews with the parents that differentiated the two groups was with regard to the stress that the parents placed upon school grades and plans for college attendance. Only one pair of the low achievers' parents said that they stressed school grades and college preparation, while eight pairs of the underachievers' parents answered this question affirmatively.

CHAPTER IV

DISCUSSION

A. Mathematics

The major mathematics objectives of this study were:

1. Formulate a flow chart for the development of concepts in two mathematical areas at grades 4-6.

2. Select three levels within the concrete-abstract continuum for each stage of the development of the concept.

3. Test the abilities of low achievers and under achievers in mathematics to function with a concept at each of three selected levels of the concrete-abstract continuum.

These objectives have been accomplished. The flow charts were developed for the areas of linear measurement and place value. Within each area three levels of the concrete-abstract continuum were identified for each stage of the development These levels were a) concrete, b) semi-concrete, and c) abstract.

The tests were administered to a group of low achievers and under achievers in mathematics from grades 4-6. The results as presented in Chapter III suggest that the students of this study do vary in their ability to function at the different levels of the concrete-abstract continuum for each mathematical area tested. That this variance in abilities depends on the mathematical area tested can be noted by analyzing the results for each group.

In the low achiever group, students 4, 10, and 11 could probably function on the abstract level for both mathematical areas; however, student 9 could probably operate on the abstract level of linear measurement and the semi-concrete on place value, while student 8 could probably operate on the abstract level of place value and semi-concrete level of linear measurement. Students 1, 6 and 7 could probably function on the semi-concrete level of place value, but only on the concrete level of linear measurement. Students 2, 3, 5 could probably function on the semi-concrete level in both areas.

Hence, it would seem that the ability to operate on the different levels of these two mathematical areas does not follow a consistent pattern for the low achievers, but rather, it is an individual problem, which must be identified for each student.

If the under achiever group is analyzed in a like manner, we find a similar pattern of variation. Students 4, 7, 9, 10, and 11 of this group could probably function on the abstract level of linear measurement and the semi-concrete level of place value, while student 12 could probably operate on the abstract level of place value and the semi-concrete level of linear measurement. Student 2 could probably function on the semi-concrete level of place value and the concrete level of linear measurement. Students 1, 3, 5 and 6 could probably function on the semi-concrete level for both mathematical areas.

As with the low achiever group, the under achiever group presents individual problems as to their abilities to function on the different levels of the continuum within these two mathematical areas, linear measurement and place value.

It would be hazardous to attempt to draw any conclusions concerning the relationship of the actual grade of the student, achievement test placement and performance on these tests However, there is some indication that if a student has an achievement test placement of fifth or sixth grade he could probably function on the abstract level of linear measurement and place value, regardless of whether he is a low achiever or under achiever in mathematics.

B. Psychological - Sociological Data

The data suggest several conclusions about the subjects in both groups of the study. First it would seem that both the low achievers and under achievers experience more emotional adjustment problems than do the typical population. This is supported by three sources — the CPI, the home interviews and the psychological observations.

Both groups attained personal, social and total adjustment scores below average on the CPI. The home interviews revealed many more "critical instances" of familial problems, and the psychologist observed many more marked adjustment problems than one would expect to find in a typical group of school children.

The entire sample of the present study is clearly biased towards the high end of the intelligence scale. One would certainly not expect to find fifteen out of twenty-three I. Q.'s at or above one-hundred-fifteen points in a random sample. The school from which the sample was drawn has a much greater number of professional and self-employed parents than is the case in most schools.

In regards to the teachers' ratings and interviews it was obvious that, in the teachers' opinions, there is a positive correlation between achievement, intelligence, and behavior. The under achievers were consistently rated higher and seen as better adjusted by the teachers than the low achievers.

Generally, reading of the parent interview protocols led the researchers to the conclusion that the quality of the family relations of the subjects was, at best, questionable. This was because of the marked ambivalence that many of the parents expressed and the large number of "critical events" encountered.

Under Achievers

The under achiever in the present study seems to be a child with personal adjustment problems, high anxiety, high ability, and good school adjustment.

From the data collected, one could present a general description of the under achiever as follows: The under achiever comes from a home that is relatively unstable, in many cases fraught with discord and disharmony. On top of this, these homes are centered around high achievement. As noted in Chapter III, a great deal of stress is placed upon school grades and college preparation. This home pressure generates a sense of a lack of personal freedom and strong anti-social tendencies, as well as a tremendous amount of anxiety.

It is easy to see how one, under such great pressure, could feel little personal freedom. The anti-social tendencies are probably a reaction to this pressure. It is as if the child were saying, "I'll do what people ask of me, but I hate everyone for asking it."

The most debilitating result, however, is clearly the anxiety. These children are anxious, and it hurts them! The more anxiety provoking the situation, the more debilitated they become. This is clear from the results of the intelligence test, as well as the fact that they are under achievers in mathematics. It is a well-known phenomenon that in our society, both mathematics and time limits are anxiety-provoking factors, even to the most well-adjusted person. The under achievers in the present study seem to be especially affected by these factors.

First, they were selected because they were under achievers in mathematics. Second, although they are far above the low achievers in terms of the untimed verbal portions of the WISC, there is no significant difference between the groups in the timed (performance) portion of the WISC, as revealed by table 5. Furthermore, when the anxiety producing factors of time and math are combined, the performance of the low and under achievers is almost identical, as revealed by table 2. When the child identified as an under achiever in the present study is confronted with high anxiety situations his performance is lowered considerably, although his basic ability is still present.

Low Achievers

The problems of the low achiever seem to be more general and pervasive than is the case with the under achiever. It is quite difficult to pinpoint anything of peculiar significance. His personal adjustment, familial relationships, school behavior, achievement and measured intelligence are below par. He has personal problems, social problems, familial problems, his teachers see him as less well adjusted than others, and his intelligence is lower than his peers, although not below average. Perhaps, had the children in the present sample been enrolled in a school that was not so heavily skewed towards the high intelligence range, they would have had less problems. It is hard to say. Nevertheless, they are having problems in every area of human endeavor in their present situation. And, somewhere, in their academic development, they have fallen significantly behind their peers.

CHAPTER V

CONCLUSIONS, IMPLICATIONS AND RECOMMENDATIONS

A. Mathematics

From an analysis of the data of this study it is recommended that the following procedure be tested as an efficient method of remediation for mathematical deficiencies of a low achiever in mathematics:

1. Locate mathematical areas of deficiency by a careful item analysis of an achievement test which is administered to each student.

2. Adminster a comprehensive diagnostic test which will cover each area of deficiency as identified by the achievement test.

3. Develop and administer tests on the three levels of the concrete-abstract continuum related to specific deficiencies as identified by the diagnostic items.

4. Develop and teach appropriate lessons on the appropriate level of the concrete-abstract continuum for each student.

From the implications of the psychological and sociological data is seems wise to recommend that efforts be made to remove the anxiety of the under achiever when he works with mathematics. Further studies should be conducted to evaluate some of the mathematical areas which may be influencing this anxiety. The development of more confidence in one's ability and the removal of threats might be a solution to this problem.

B. Psychological-Sociological

The entire group of subjects in this sample both under and low achievers are having personal adjustment problems that are interfering with their academic achievement. It is evident that they would all benefit from a program of personal counseling. Therefore, the first and foremost recommendation would be that low and under achievers, such as those in the present study, receive a well organized program of counseling. However, since a good counseling program is often unavailable, recommendations must be considered that can be carried out in the classroom by a classrom teacher. Because of the characteristics of the subjects as revealed by the data, different recommendations are in order for the two groups..

Low Achievers

The low achiever in the present study needs a great deal of assistance in many areas - personal, social, and academic. But his problems are so pervasive that they defy amilioration in every respect by the school system. They need intensive counseling as well as intensive educational tutoring. Probably the best thing a school could do for them is to involve them in some kind of comprehensive remedial program to help them in all academic areas as well as in mathematics. Such a program might help them, as well, in some of the non-academic problem areas.

Under Achievers

The problems of the under achievers in the present study seem to be, in some respects, more specific. If the anxiety of these children could in any way be reduced, their performance, if not their personal-social adjustment would improve. Consequently, the following suggestions are made in regards to the under achievers:

1. The under achiever might be placed in a regular class, but with his achievement being measured only at the end of the year rather than constantly throughout the year. This might lower the dibilitating anxiety.

2. The under achiever might be placed in a special class in which emphasis is placed upon the "usefulness" and "enjoyment" of mathematics, rather than grades. His achievement could then be measured at the end of the course or year.

3. Allow the under achiever to tutor the low achievers in his class, and grade the under achiever either on (a) how well the low achiever improves, (b) or on the basis of the under achiever's own achievement at the end of the year. It is possible that this procedure would enhance the under achiever's self-concept by allowing him to be a "teacher" and relieve the anxiety of being directly and consistently assessed. Such a procedure might improve his academic performance as well as his personal-social adjustment.

4. Further studies should be conducted in an attempt to identify the specific mathematics, if any, which might be causing the high anxiety levels. It is possible that learning to work with mathematics with confidence might remove some of the anxiety.

CHAPTER VI

SUMMARY

It was the purpose of this study to explore factors related to under achievement and low achievement in mathematics education and to determine if there are individual levels of abilities in abstractive thought with respect to mathematics concepts.

The objectives of the study were to:

1. Formulate a flow chart for the development of concepts in two maheematical areas at grades 4 - 6.

2. Select three levels within the concrete-abstract continuum for each stage of the development of the concepts.

3. Test the abilities of a group of low achievers and under achievers in mathematics to function with a concept at each of three selected levels of the concrete-abstract continuum.

4. Make an intensive analysis of each student using the case study technique.

Twelve under achievers and eleven low achievers from a middle class school comprised the sample. A subject was considered an under achiever in mathematics if his non-mathematics scores were equal to or above his present grade placement, but his average percentile score in mathematics computation and concepts was two or more deciles below his non-mathematics percentile average.

A subject was considered a low achiever if his average percentile scores on all sections of the achievement test were at least two deciles below his present grade placement level.

In addition, the subjects chosen were at least average measured intelligence or above. This factor was defined in terms of Wechsler's (1958, p. 42) table of I.Q. distributions.

Data was obtained in mathematics from tests developed by the staff on three abstract levels (a) concrete, (b) semi-concrete, (c) abstract. These levels were established for the areas of linear measurement and place value. A sequence of concepts from each of these areas was developed and items for each level were formulated.

Intensive case studies were conducted for each student. Objective data was collected from Cumulative Records, California Personality Inventory, Wechsler Intelligence Scale for Children, and Teachers Rating Scale for Pupil Adjustment.

Subjective data was obtained from Psychological Observation, How I See Myself Essay, Teacher Interviews and Parent Interviews.

The results of the study can be summarized as group data, however the particular strength of this technique of analysis is its ultimate value in helping individuals.

In the low achiever group, three (3) students could probably function at the abstract level for both mathematical areas. Three (3) students could probably function on the semi-concrete level of place value and on the concrete level of linear measurement. Three (3) students could probably function on the semi-concrete level in both areas. One (1) student could probably function on the abstract level of linear measurement and the semi-concrete level of place value, while one (1) student could probably operate on the abstract level of place value and the semi-concrete level of linear measurement.

In the under achiever group, five (5) students could function on the abstract level of linear measurement and the semi-concrete level of place value. Four (4) students could probably function on the semi-concrete level for both areas. One student from this group could operate only on the concrete level of linear measurement.

It would appear that the ability to operate on the different levels of these two mathematical areas does not follow a consistent pattern for either group. Hence, it is an individual problem which must be considered for each student.

The psychological and sociological data suggest that both the low achievers and under achievers experience more emotional adjustment problems than do the typical population. Both groups attained personal, social, and total adjustment scores below average on the California Personality Inventory Test. The home interviews revealed many more "critical instances" of familial problems, and the psychologist observed many more marked adjustment problems than could be expected in a typical group of school children.

The teachers rated the under achievers consistently higher in achievement and adjustment than the low achievers. From the parent interviews, the quality of the family relations of the subjects was questionable. The parents indicated marked ambivalence in their relationships with the subjects.

The under achiever of this study comes from a home that is relatively unstable, in many cases fraught with discord and disharmony. On top of this, these homes are centered around high achievement. A great deal of stress is placed upon school grades and college preparation. This home pressure generates a sense of a lack of personal freedom and strong anti-social tendencies, as well as a tremendous amount of anxiety. When the under achiever of this study was confronted with high anxiety situations his performance was lowered considerably.

The low achiever of this study is more difficult to analyze. His personal adjustment, familial relationships, school behavior and achievement are below par. They are having problems in every area of human endeavor in their present situation,and somewhere they have fallen significantly behind their peers in their academic development.

From an analysis of the data of this study it is recommended that the low achievers be involved in a comprehensive program of intensive counseling and a remediation program in mathematics based upon their individual levels of abstractive ability.

The under achievers should be involved in programs which will reduce their anxiety toward mathematics. If this anxiety can be reduced, their performance in mathematics would probably improve and possibly their personal - social adjustment also would improve.

REFERENCES

1. Carrothers, George E. "Why do High School Pupils Fail?" National Association of Secondary School Principals Bulletin, 30: 1946, pp. 29-36
2. Cook, Edward S. "An Analysis of Factors Related to Withdrawal from High School Prior to Graduation," Journal of Educational Research, 50:190-96; 1956
3. Counts, George S. The Selection Character of American Secondary Education. University of Chicago, 1922, p. 162.
4. Cummings, Howard H. and others. A Look Ahead in Secondary Education. U. S. Office of Education Bulletin, 1954, No. 4, Government Printing Office, p. 105.
5. Delaney, John F. "That Vacant High School Seat," American School Board Journal, 121, Nov. 1950, pp.22-23.
6. Dillon, Harold J. Early School Leavers: Major Educational Problem. National Child Labor Committee, 1949, p. 95.
7. Durrell, Donald D. "Learning Difficulties Among Children of Normal Intelligence," Elementary School Journal, 55: 1954, pp.201-8.
8. Edelston, Harry. "Education Failure with High Intelligence Quotient: A Clinical Study," Journal of Genetic Psychology, 77:1950, pp.85-116
9. Farnsworth, Burton K. and Casper, J. B. "A Study of Pupil Failure in High School," School Review, 49:1941, pp.380-83.
10. Karpinos, Bernard D. and Sommers, Herbert J. "Educational Attainment of Urban Youth in Various Income Classes," Elementary School Journal, 42: 677-78, 767-74; 1942.
11. Lafferty, Harry W. "Reasons for Pupil Failure," American School Board Journal, 117: 1948, pp. 18-20.
12. Penty, Ruth C. Reading Ability and High School Drop-Outs. Twentieth Century Fund, Inc. 1956, p.93.
13. Shontz, Franklin C., Research Methods in Personality. New York: Appleton-Century-Crofts, 1965.

14. Strom, Robert D. "The School Drop-Out and the Family," School and Society, 92:191-92, April 19, 1964.

15. Watson, R. I., The Clinical Method in Psychology, New York: Harper, 1951, pp. 68-71.

16. Wechsler, David, Wechsler Intelligence Scale for Children, Manual. New York: The Psychological Corporation, 1949, 114 pp.

17. Whaley, Charles E., "Youngsters Who Quit School," Kentucky School Journal, 42: 17, January 1964.

Appendix A

Linear Measurement
Concrete

Name______________

I Three sticks -- different lengths and color

(1) __________ ________ _____
blue green red

PICK UP THE RED STICK!	colors identified correctly yes__no__
PICK UP THE BLUE STICK!	if no, comment________
PICK UP THE GREEN STICK!	

(2) If colors identified correctly,

WHAT COLOR IS THE SHORTEST STICK?	red_________ blue_____
WHAT COLOR IS THE LONGEST STICK?	red_________ blue_____

Do item 3 only if 2 is not correct.

(3) Same sticks

HAND ME THE SHORTEST STICK!	red_________ blue_____
HAND ME THE LONGEST STICK!	red_________ blue_____

II (Show student one stick.)

WOULD YOU CALL THIS STICK A LONG STICK?	long__________________ short_________________ other response________

III (1) Three sticks, 4", 6", 7", with the 7" stick glued to underside of a cardboard 8" x 6", with the 4" and 6" sticks glued to top of cardboard.

(Point to 6" stick and state),

THE STICK UNDER THE CARDBOARD IS LONGER THAN THIS STICK.

(Point to the 4" stick and ask),

IS THE STICK UNDER THE CARDBOARD LONGER OR SHORTER THAN THIS STICK?

longer______________

shorter______________

can't tell___________

(2) Bring out a card with a 5" stick glued to the underside.

State,

THE STICK UNDER THIS CARD IS SHORTER THAN THIS STICK. (Point to 6" stick.)

IS IT SHORTER OR LONGER THAN THIS STICK? (Point to 4" stick.)

longer______________

shorter______________

can't tell___________

IV (1) Two sticks -- different lengths

Show the student they are different.

Cover part of both sticks with a cardboard with visible ends matching.

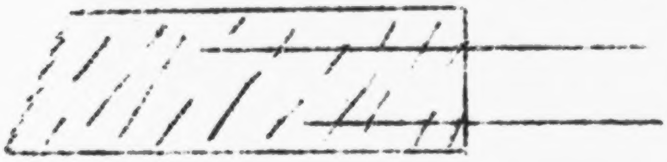
POINT TO THE LONGER STICK.

makes any choice_____

removes paper yes__no__

can't tell__________

no response__________



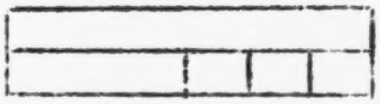
(2) If either stick is chosen, ask,

HOW DO YOU KNOW?

response ____________

____

V (1) One six-unit stick; one three-unit, three one-unit

Place 

Now remove six-unit stick and mix up others. Not in line!

I HAVE REMOVED THE LONG STICK. SHOW ME HOW LONG IT WAS!

observation _________

____

(2) Move one shorter stick to the other end.

IS THIS THE SAME LENGTH AS THE STICK I REMOVED?

yes________ no______

response____________

VI (1) (Hand student 2 red rods of equal length.)

WHAT IS TRUE ABOUT THE LENGTH OF THESE STICKS?

same__________________
don't know____________
no response___________

(2) Place gold rod (equal 2 reds) on table with reds. Don't line up!

WHAT DO YOU NOTICE ABOUT THE LENGTHS OF THESE STICKS?

lines them up yes__no__
red is shorter________
gold is longer________
no response___________
r and r = gold________

(3) If "red is shorter" and/or "gold is longer" is a response, ask,

DO YOU NOTICE ANYTHING ELSE?

lines them up yes__no__
no response___________
r and r = gold________
other_________________

VII (Hand student colored sticks.)

3 units - green
2 units - red
1 unit - white

TELL ME ALL YOU CAN ABOUT THEIR LENGTHS!

lines them up yes__no__
green longest_________
white shortest________
red + white = green___
other_________________
sketch the "line-up"

VIII Lay out uncolored stick 10" in length. Use 4 - 5" reds, 7 - 2" blues, 5 - 3" greens.

(1) HOW MANY REDS HAVE THE SAME LENGTH AS THIS STICK?

don't know____________

2 reds________________

uses 2 reds___________

uses 1 red twice______

(2) HOW MANY BLUES HAVE THE SAME LENGTH?

don't know____________

5 blues_______________

uses 5 blues__________

uses 1 blue 5 times___

(3) HOW MANY GREENS HAVE THE SAME LENGTH?

can't tell____________

approximately_________

(4) EXPRESS IN AS MANY WAYS AS YOU CAN, USING COLORS, THE LENGTH OF THE UNCOLORED STICK.

2 reds________________

red + blue + green____

2 blue + 2 green______

5 blues_______________

sketch any other ways

IX Give 12" ruler (standard) and 3 blocks, 8", 6", 4"

MEASURE EACH OF THE STICKS AND TELL ME THE LENGTHS!

uses zero as origin___

uses other point as origin____________

measures correctly____

gives length as "about"______________

X (1) Unroll a 12" (estimate) strip of cash register tape from the student's left. Fasten the right end to the table with masking tape. Hand the student a marking pencil, a unit stick, and a stick to measure.

WE WANT TO MEASURE THIS STICK.

MAKE A RULER USING THIS PAPER. A BLOCK MAY REPRESENT 1 GISMO OF LENGTH.

PLEASE LABEL.

can't start__________

makes a zero point
at end______________
not at end__________

does not mark zero____

labels points_________

labels intervals______

uses block each time__

(2) Cut off the tape with a scissors about two units before the first marked unit.

WE WANT TO KNOW HOW MANY GISMO'S LONG THIS STICK IS. CAN YOU USE YOUR RULER AND FIND OUT?

starts with___________

adjusts_______________

doesn't adjust________

counts units
(doesn't use scale)__

can't measure_________

(3) Cut off the tape with a scissors at the "two unit" mark. Give the student a different stick.

HOW MANY GISMO'S LONG IS THIS STICK?

If correct, ask,

HOW DID YOU FIND OUT?

starts with___________

adjusts_______________

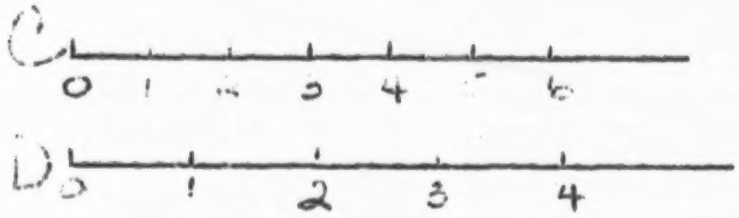
doesn't adjust________

counts units, not
scale_______________

subtracts_____________

can't measure_________

XI Use rulers C and D and measure stick (3")



(1) FIND THE LENGTH OF THE STICK USING THIS RULER! (C)

(2) FIND THE LENGTH OF THE STICK USING THIS RULER! (D)

(3) WHAT IS THE RELATIONSHIP OF THE UNITS OF THE TWO RULERS?

measures C as 6______

measures D as 4______

comments on different scales______

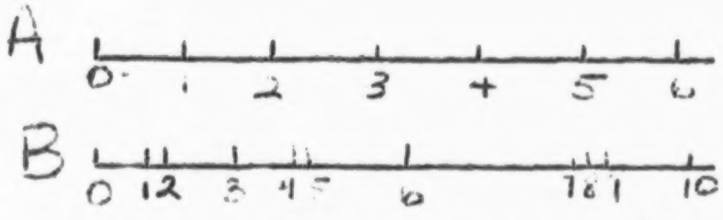
not disturbed______

anxiety shown______

response ______

______

XII Hand ruler A to student and have him measure block (6"), then ruler B.



(1) FIND THE LENGTH OF THE STICK!

(2) FIND THE LENGTH OF THE STICK WITH THIS RULER! (B)

(3) IS RULER B A GOOD RULER? WHY?

measures A as 3______

measures B as 6______

comments on non-uniform units______

not disturbed by rulers______

anxiety shown______

response ______

______

XIII Board with two nails 10" apart and a handful of inch-sticks.

Have piece of unmarked plexiglas straight edge handy for alignment.

Point to nails.

THESE ARE INCH-STICKS. HOW MANY INCHES FROM THIS NAIL TO THIS NAIL? (Point to nails.)

correct____________

unable to measure the distance________

uses edger__________

XIV 15 - 1" sticks
15" stick on piece of paper
18" rule marked in 3" units

Hand student paper and rule.

(1) THIS IS AN 18" RULE MARKED IN 3" UNITS.

HOW LONG IS THAT STICK?

15" (correct)________

no response__________

incorrect___________

(2) If response to 1 is correct, hold inch sticks in hand; place 2 sticks end to end on left edge of line to student.

EACH OF THESE STICKS IN MY HAND IS ONE INCH LONG. LET'S CALL THEM INCH STICKS. HOW MANY OF THEM WOULD WE NEED IF WE PLACED THEM END TO END ON THE STICK FROM HERE TO HERE?

(Examiner, place a finger at each end of the stick.)

15______________

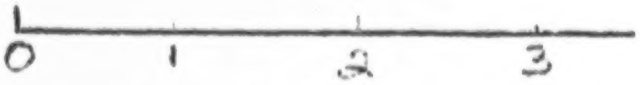
don't know__________

other______________

hesitation yes___no__

memory lapse __secs.

XV Marked ruler with submarkings (12" or 18" long) -- handful of inch sticks



Lay down a 1st and 2nd inch stick end to end beside the ruler.

IF WE CONTINUE TO PLACE THE INCH STICKS, WHERE WILL YOU PUT THE TENTH INCH STICK? PLACE IT IN POSITION!

guesses_______________

uses ruler____________

placement:

9 - 10 interval______

9 1/2 - 10 1/2 interval____________

10 - 11 interval_____

other position_______

XVI Sticks 7-0/16" , 8-0/16" , 6-4/16" , 7-11/16" (red)

2 - 12" rulers, one with marks each inch, one with inches marked and 1/4" subdivisions

(1) Place 6-4/16" stick down. Give student 1/4" ruler.

HOW LONG IS THE STICK?

6"________________

6-14"______________

other______________

(2) Lay the 7-0/16" and 8-0/16" sticks beside the 7-11/16" (red) stick. Point to 7-0/16" and 8-0/16" sticks.

THE RED STICK IS NEARER IN LENGTH TO THIS STICK OR THIS STICK?

7-0/16"____________

8-0/16"____________

don't know__________

(3) Use 7-11/16" stick (red). Hand student ruler marked in inches only.

USING THIS RULER YOU WOULD SAY THE LENGTH OF THE RED STICK IS 7" OR 8"?

7"________________

8"________________

other______________

no answer___________

hesitates___________

(4) Same red stick. Hand student ruler with 4/16" subdivisions.

USING THIS, WHAT WOULD BE YOUR MEASURE OF THE RED STICK?

response____________

hesitation__________

no answer___________

Appendix B
Linear Measurement
Semi-Concrete

Name ____________________

I (1) THESE ARE PICTURES OF A TOOTHPICK (point to pictures), BALLPOINT PEN, AND BASEBALL BAT.

WHICH OF THE PICTURED THINGS WOULD BE THE SHORTEST?

toothpick____________
pen________________
bat________________

(2) WHICH OF THE PICTURED THINGS WOULD BE THE LONGEST?

toothpick____________
pen________________
bat________________

II WOULD YOU CALL THIS A LONG BOX?

yes________________
no_________________
other______________

III (Point to pictures of objects)

(1) THE BLACK BOX CONTAINS AN OBJECT THAT IS LONGER THAN THE PEN. IS THE OBJECT IN THE BLACK BOX LONGER THAN THE NAIL?

yes________________
no_________________
other______________

(2) THE RED BOX CONTAINS AN OBJECT THAT IS SHORTER THAN THE PEN. IS THE OBJECT LONGER THAN THE NAIL?

yes________________
no_________________
can't tell__________
other______________

IV (1) THIS IS A PICTURE OF TWO POSTS DIRECTLY BEHIND A WALL. THE POSTS ARE NOT THE SAME LENGTH.

WHICH IS LONGER, POST A OR POST B?

A__________________
B__________________
can't tell__________
no response_________

(2) (If either post chosen),

HOW DO YOU KNOW?

response

V (Place the stick next to each of the colored block strips. Remove stick.) (1) WHICH SET OF COLORS REPRESENTS THE LENGTH OF THE STICK WHICH I HAD SHOWN TO YOU?	1 r + 3 b____________ 2 r + 2 b____________ 2 g______________ other____________
(2) (If answer to 1 is correct, turn the acetate) IS IT STILL THE LENGTH OF THE STICK?	yes______________ no______________ other____________
VI (Cover the bottom part of acetate.) (1) WHAT IS TRUE ABOUT THE LENGTH OF THESE BLOCKS?	same______________ doesn't know__________ no response__________
(2) (Uncover bottom of acetate.) WHAT DO YOU NOTICE ABOUT THE LENGTH OF THESE BLOCKS?	g > r______________ g > b______________ r + b = g___________ no response__________ other______________
(3) (If r + b = g not given), DO YOU NOTICE ANYTHING ELSE?	r + b = g___________ no response__________ other______________
VII TELL ME ALL YOU CAN ABOUT THE LENGTHS OF THESE BLOCKS.	black longest_________ red shortest_________ blue + red = black____ blue = 2 red_________ black = 3 red________ other______________

VIII (Point to picture of blocks.)

(1) THESE ARE A GREEN BLOCK, A BLUE BLOCK, A RED BLOCK, AND AN ORANGE BLOCK.

HOW MANY BLUES HAVE THE SAME LENGTH AS THE GREEN BLOCK?

two______________________
other____________________

(2) HOW MANY REDS HAVE THE SAME LENGTH AS THE GREEN BLOCK?

five_____________________
other____________________

(3) HOW MANY ORANGE BLOCKS HAVE THE SAME LENGTH AS THE GREEN BLOCK?

can't tell_______________
approximately____________

(4) EXPRESS IN AS MANY WAYS AS YOU CAN, USING COLORS, THE LENGTH OF THE GREEN BLOCK.

2 b______________________
5 r______________________
b + r + o________________
2 r + 2 o________________
other____________________

IX (Place plastic ruler on acetate incorrectly.)

(1) I WANT TO MEASURE THE LENGTH OF THIS GREEN BLOCK. TELL ME HOW TO PLACE THE RULER TO MEASURE IT.

response

(2) HOW LONG IS THE BLOCK?

correct__________________
other____________________

X (Point to unmarked block.)

HOW LONG IS THIS BLOCK?

6________________________
7________________________
other____________________
counts intervals_________

XI	(1)	WHAT IS THE LENGTH OF THE LINE SEGMENT AB USING THE RED RULER?	correct_____________ other_______________
	(2)	WHAT IS THE LENGTH OF AB USING THE BLUE RULER?	correct_____________ other_______________
	(3)	WHAT IS THE RELATIONSHIP OF THE SIZE OF THE UNITS OF THE TWO RULERS?	1 r = 2 b____________ 1 b = 1/2 r__________ other_______________ no response_________
XII	(1)	WHAT IS THE LENGTH OF AB USING THE RED RULER?	correct_____________ other_______________
	(2)	WHAT IS THE LENGTH OF AB USING THE BLUE RULER?	correct_____________ other_______________
	(3)	IS THE RED RULER A GOOD RULER? WHY?	yes_________________ no__________________ comments____________
XIII		(Point to picture of the red block as question is asked.) USING THIS ONE-INCH RED BLOCK, HOW COULD YOU DETERMINE THE NUMBER OF INCHES FROM A TO B?	response____________

XIV (Show ruler on overhead, remove, and place on top of acetate having blue block.)

(1) THIS IS A NINE-INCH RULER MARKED IN THREE-INCH UNITS. I WANT TO MEASURE THE BLUE BLOCK. TELL ME HOW TO PLACE THE RULER.

response

(2) HOW LONG IS THE BLUE BLOCK?

correct______________

other______________

(3) (If response to 2 is correct, place red block on acetate and remove ruler.)

THIS RED BLOCK IS AN INCH IN LENGTH. HOW MANY RED BLOCKS WOULD WE NEED IF WE PLACED THEM END TO END ON THE BLUE BLOCK TO COVER THE BLUE BLOCK.

six________________

don't know___________

other______________

hesitates yes___no___

memory lapse ___secs.

XV (Place inch blocks on first and second intervals.)

IF I CONTINUE TO PLACE THE INCH-BLOCKS WHERE SHOULD I PLACE THE TENTH INCH-BLOCK?

NAME THE TWO COLORS THE BLOCK WOULD COVER.

red and blue_________

blue and green________

green and black_______

black and blue________

XVI (Place acetate on projector.)

(1) HOW LONG IS THE RED BLOCK?

6________________
6-1/4________________
other________________

(2) (Remove acetate. Place colored strips on projector. Line up strips.)

THE RED BLOCK IS NEARER IN LENGTH TO WHICH BLOCK, BLUE OR GREEN?

green________________
blue________________
doesn't know__________

(3) (Remove blue and green strips. Lay inch ruler next to red strip.)

IS THE LENGTH OF THE RED BLOCK NEARER TO 7 INCHES OR 8 INCHES?

7________________
8________________
other________________
no answer____________
hesitates____________

(4) (Remove inch ruler. Lay 1/4" ruler next to red strip.)

USING THIS RULER MARKED IN 1/4 INCHES, WHAT WOULD BE THE MEASURE OF THE RED BLOCK?

response____________
____
hesitation____________
no answer____________

Appendix C
Linear Measurement
Abstract

Name________________

I WE HAVE A BASEBALL BAT, A TOOTHPICK, AND A BALLPOINT PEN.

(1) WHICH ONE OF THESE IS THE SHORTEST?

(2) WHICH ONE IS THE LONGEST?

(1) bat________________
toothpick____________
pen________________
other______________

(2) bat________________
toothpick____________
pen________________
other______________

II A MAN GOES TO A STORE TO BUY A CANE FOR WALKING. THE STORE HAS CANES OF MANY DIFFERENT LENGTHS. HE BUYS ONE OF THE CANES.

DID HE BUY A LONG CANE?

long________________
short_______________
other_______________

III (1) BILL WENT TO A HARDWARE STORE TO BUY NAILS. HE WAS SHOWN SOME WHICH WERE LONGER THAN HIS BALLPOINT PEN.

WERE THESE NAILS SHORTER OR LONGER THAN A TOOTHPICK?

longer______________
shorter_____________
can't tell__________

(2) BILL ALSO WAS SHOWN A NAIL WHICH WAS SHORTER THAN THE BALLPOINT PEN.

WAS THIS NAIL SHORTER OR LONGER THAN A TOOTHPICK?

longer______________
shorter_____________
can't tell__________

IV BILL AND JIM DROVE TWO POSTS OF DIFFERENT LENGTHS INTO THE GROUND AND NAILED A BOARD ACROSS THE TOP OF THE POSTS.

IF THE BOARD IS LEVEL ON THE POSTS, WHICH POST WAS LONGER?

can't tell__________
other_______________

V (1) SOME BOYS WANTED TO MOVE A BENCH FROM THE LOCKER ROOM TO THE DUG-OUT AT THE BASEBALL FIELD. THEY WEREN'T SURE WHETHER OR NOT THE BENCH WAS TOO LONG. IN THE LOCKER ROOM JOE FOUND THAT THE BENCH WAS THREE BASEBALL BATS AND ONE BALL LONG.

HOW COULD JOE SHOW THE OTHER TEAM MEMBERS ON THE FIELD HOW LONG THE BENCH WAS WITHOUT TAKING THE BENCH TO THE FIELD?

response_______________

____

____

____

(2) WHICHEVER WAY PART 1 IS ANSWERED, RESTATE THE LENGTH WITH THE BALL IN A DIFFERENT PLACE. SUCH AS: IF THE ANSWER TO 1 IS "3 BATS AND 1 BALL," THEN ASK,

IS IT 2 BATS, A BALL, AND A BAT?

yes____________________

no_____________________

other__________________

VI (1) PAUL AND BILL ARE PLAYING DARTS IN THE BOYS' CLUB. THE CLUB MANAGER TELLS PAUL THAT HE CAN TAKE THE DARTS AND THE TARGET HOME WITH HIM TO PLAY ONE NIGHT. THEY NOTICE THAT THE DISTANCE THEY ARE SUPPOSED TO STAND FROM THE TARGET TO THROW THE DARTS IS EQUAL TO THE COMBINED LENGTHS OF THEIR ARM SPANS. PAUL'S AND BILL'S ARM SPANS ARE THE SAME.

HOW CAN PAUL ALONE DETERMINE HOW FAR FROM THE TARGET TO STAND WHEN HE MOUNTS IT IN HIS GARAGE?

correct______________
other______________

(2) SUPPOSE WE HAVE TWO STICKS, A RED AND A YELLOW, WHICH ARE OF EQUAL LENGTH.

SUPPOSE WE FIND A GREEN STICK WHICH IS EQUAL IN LENGTH TO THE LENGTH OF YELLOW PLUS THE LENGTH OF THE RED.

WHAT IS THE RELATIONSHIP OF THE LENGTH OF THE GREEN STICK AND THE RED STICK?

$r + r = g$______________
$g = 2r$______________
$r = 1/2g$______________
other______________

VII SUPPOSE WE FIND A YELLOW STICK WHOSE LENGTH IS EQUAL TO THE LENGTH OF THE BLUE STICK PLUS THE LENGTH OF A RED STICK.

THE BLUE STICK'S LENGTH IS EQUAL TO THE LENGTH OF TWO RED STICKS.

WHAT IS THE RELATIONSHIP OF THE LENGTHS OF THE YELLOW STICK AND THE RED STICK?

$y = 3 r$ ________________
$y = 1/3 y$ ________________
$y > r$ ________________
other ________________

VIII (1) WE HAVE A WHITE STICK WHICH IS 10 UNITS LONG AND RED STICKS WHICH ARE 5 UNITS LONG.

HOW MANY REDS EQUAL THE LENGTH OF THE WHITE STICK?

two ________________
other ________________

(2) IF WE HAVE BLUE STICKS WHICH ARE 2 UNITS LONG, HOW MANY BLUES EQUAL THE LENGTH OF THE WHITE STICK WHICH WAS 10 UNITS LONG?

five ________________
other ________________

(3) IF A GREEN STICK IS 3 UNITS LONG, HOW MANY GREEN STICKS EQUAL THE LENGTH OF THE WHITE STICK?

can't tell ____________
3-1/3 ________________
other ________________

(4) USING COLORED STICKS,
RED - 5 UNITS
BLUE - 2 UNITS
GREEN - 3 UNITS
HOW MANY WAYS CAN YOU EXPRESS THE LENGTH OF THE WHITE STICK WHICH IS 10 UNITS LONG?

2 reds ________________
$r + b + g$ ________________
$2 b + 2 g$ ________________
$5 b$ ________________
other ________________

IX BILL IS MEASURING HIS HEIGHT BY PLACING A MARK ON THE WALL WHERE THE TOP OF HIS HEAD TOUCHES. HE THEN USES A TAPE MEASURE TO FIND THE DISTANCE FROM THE FLOOR TO THE MARK. HE MEASURES HIS HEIGHT AS 64 INCHES, BUT DISCOVERS THAT THE FIRST 3 INCHES OF THE TAPE IS TORN OFF. HOW TALL IS BILL?

correct________________

67____________________

other_________________

X JOHN MEASURES THE LENGTH OF A STICK WITH A 12" RULER WHICH HAS 3 INCHES BROKEN OFF EACH END.

IF THE END OF THE STICK LIES ON THE SIX OF THE RULER WHEN THE OTHER END OF THE STICK LIES AT THE END OF THE RULER, WHAT IS THE LENGTH OF THE STICK?

correct________________

six___________________

other_________________

XI BILL AND JIM EACH USE A DIFFERENT RULER TO MEASURE A GIVEN STICK. BILL REPORTED THE LENGTH OF THE STICK WAS 6 UNITS AND JIM REPORTED THE LENGTH OF THE STICK WAS 12 OF HIS UNITS. WHOSE UNIT OF MEASUREMENT WAS LARGER, BILL'S OR JIM'S?

Bill's_________________

Jim's__________________

XII ALICE AND BILL ARE TRYING TO DETERMINE HOW MANY STEPS IT IS FROM BILL'S HOME TO SCHOOL. BILL WALKS 100 STEPS, RUNS 500 STEPS AND JUMPS 20 STEPS TO ARRIVE AT SCHOOL. HE ANNOUNCES THAT IT IS 620 STEPS FROM HIS HOUSE TO SCHOOL!

IS THERE ANYTHING WRONG WITH HIS CONCLUSION?

Response

		Response
XIII	(1) JIM HAS A BASEBALL BAT WHICH IS 3 FT. IN LENGTH. HE WISHES TO DETERMINE THE DISTANCE FROM HOME PLATE TO FIRST BASE ON A BASEBALL DIAMOND. HOW CAN HE DETERMINE THIS DISTANCE USING THE BAT?	
	(2) IF HE FINDS THAT THERE ARE 30 LENGTHS OF THE 3 FT. BAT, HOW FAR IS IT FROM HOME PLATE TO FIRST BASE?	10__________ 90__________ other__________
XIV	A MAN WISHES TO MAKE A TOP FOR A WALL USING BLOCKS PLACED END TO END. IF THE BLOCKS ARE 1 FOOT IN LENGTH AND THE WALL IS 15 FT. IN LENGTH, HOW MANY BLOCKS ARE NEEDED?	correct__________ other__________
XV	A BOY IS LAYING 1 INCH STICKS END TO END ALONG A RULER. IF HE LAYS THE FIRST STICK WITH LEFT END AT ZERO AND THE RIGHT END AT ONE, WHERE SHOULD HE PLACE THE RIGHT END OF THE TENTH INCH STICK?	9__________ 10__________ 11__________ other__________
XVI	SUPPOSE WE HAVE A STICK WHICH IS 6-3/4 INCHES IN LENGTH.	
	(1) IS ITS LENGTH NEARER TO 6 INCHES OR 7 INCHES?	(1) 6__________ 7__________ can't tell__________
	(2) IS ITS LENGTH NEARER TO 6-1/2 INCHES OR 7 INCHES?	(2) 6-1/2__________ 7__________ can't tell__________
	(3) IS ITS LENGTH NEARER TO 6-11/16 INCHES OR 6-15/16 INCHES?	(3) 6-11/16__________ 6-15/16__________ can't tell__________

Appendix D
Place Value
Concrete

Name____________________

I	(1)	(Give student 22 sticks.) COUNT THESE STICKS!	counts by ones________ other________________ makes error__________
	(2)	(Lay out 5 bundles of tens.) HOW MANY BUNDLES OF STICKS ARE THERE?	five_________________ doesn't know_________ hesitates____________
	(3)	(Leave 5 bundles of tens.) IF EACH BUNDLE CONTAINS TEN STICKS, HOW MANY STICKS ARE THERE?	fifty________________ five bundles of ten___ doesn't know_________ hesitates____________
II	(1)	(Lay out 12 bundles of tens.) HOW MANY BUNDLES ARE THERE?	correct______________ other________________
	(2)	EACH BUNDLE CONTAINS TEN STICKS. HOW MANY STICKS ARE THERE?	correct______________ other________________
	(3a)	(If correct on 2, say) HOW DO YOU KNOW?	10 x 12 = 120________ add zero_____________ counts by tens_______ doesn't know_________
	(3b)	(If incorrect on 2, say) COUNT THE STICKS BY TENS!	correct______________ pauses at 100________

III (Lay out 4 bundles of tens.)

EACH OF THESE BUNDLES CONTAINS TEN STICKS.

(Lay out 3 sticks.)

forty-three__________

HOW MANY STICKS ARE THERE?

4 tens and 3 ones_____

other_________________

(Wait for oral response.)

WRITE THE NUMERAL!

numeral_______________

IV (Lay out two piles of sticks -- 83 -- one bundled in tens, the other unbundled.)

EACH PILE OF STICKS CONTAINS THE SAME NUMBER OF STICKS! THE BUNDLES CONTAIN TEN STICKS EACH! YOU MAY COUNT THE STICKS IN EITHER PILE.

counts the bundled pile__

by 10, 20, 30__________

by 1, 2, 3_____________

counts unbundled______

HOW MANY STICKS ARE THERE IN EACH PILE? COUNT ALOUD!

counts both__________

other________________

V (Lay out 16 tens and 3 ones.)

(1) THERE ARE TEN STICKS IN EACH OF THESE BUNDLES.

163__________________

16 tens, 3 ones_______

HOW MANY STICKS ARE THERE HERE?

other________________

(2) REMOVE 100 STICKS FROM THE PILE.

correct______________

counts bundles________

(3) HOW MANY STICKS ARE LEFT?

sixty-three___________

counts again yes__no__

VI (Have plastic numerals and bundled sticks available. Show student 2 one hundred bundles; 3 tens bundles; and 4 ones.)

(1) IF THESE BUNDLES EACH CONTAIN 100 STICKS AND THESE CONTAIN 10 STICKS EACH, USE THE PLASTIC PIECES TO FORM A NUMERAL WHICH WILL TELL HOW MANY STICKS ARE IN FRONT OF YOU!

forms 234____________
other arrangement_____
hesitates yes___no___

(2) (If 1 correctly done, pick up 2 more tens bundles and show student. Hand student circle frame.)

PLACE THE FRAME OVER THE ONE SYMBOL YOU WOULD CHANGE, IF I LAY OUT TWO MORE BUNDLES OF TENS!

places frame over
4 3 2
doesn't know__________

(3) (If 2 correctly done),

MAKE THAT CHANGE SO THAT THE NUMERAL REPRESENTS CORRECTLY THE NUMBER OF STICKS NOW IN FRONT OF YOU.

changes 3 to 5________
other_________________
can't make change_____

(4) (Pick up the frame.)

PLACE THE FRAME OVER THE ONE SYMBOL YOU WOULD CHANGE IF I GIVE YOU 5 MORE STICKS!

(Lay out 5 more sticks.)

places frame over
4 5 2
can't do it___________

(5) MAKE THAT CHANGE SO THAT THE NUMERAL REPRESENTS CORRECTLY THE NUMBER OF STICKS NOW IN FRONT OF YOU!

changes 4 to 9________
other_________________
can't make change_____

VI (continued)

(6) (Pick up the frame. Numeral is now 259. If student has this, then continue.)	
IF I GIVE YOU 3 MORE STICKS, PLACE THE FRAME OVER THE ONE SYMBOL WHICH YOU WOULD CHANGE! (Lay out 3 more sticks.)	places over 5 and 9___ places over 9 5 2 student comment or reaction____________
(7) MAKE THAT CHANGE ON THE NUMERAL.	changes correctly to 262________________ other________________
(8) CAN YOU REBUNDLE THE STICKS SO THAT THE NUMERAL CORRECTLY REPRESENTS THE NUMBER IDEA? DO SO!	rebundles correctly___ doesn't understand____ other________________

VII (1) (Have boxes: box of ones; box of tens; box of hundreds.)

THIS BOX CONTAINS BUNDLES OF 100 STICKS, THIS BOX TENS BUNDLES, AND THIS ONES.

(Point to boxes. Place plastic numerals before student as each part is asked.)

CHOOSE STICKS FROM THE BOXES THAT WILL ILLUSTRATE THE MEANING OF THIS NUMERAL!

(Repeat statement for each of these numerals.)

(a) 5

(b) 14

(c) 22

(d) 206

(e) 353

correctly illustrates

5____________________

14___________________

22___________________

206__________________

353__________________

comments______________

(2) (Change the 5 to 2 in e above.)

NOW MAKE THE STICKS SHOW THIS NUMBER OF STICKS.

correct_______________

recounted_____________

other_________________

VIII (Use boxes of hundreds, tens and ones.)

THIS BOX CONTAINS BUNDLES OF 100 STICKS, THIS BOX TENS BUNDLES, AND THIS ONES. I WILL PLACE STICKS IN FRONT OF YOU AND I WANT YOU TO FORM A NUMERAL USING THE PLASTIC SYMBOLS WHICH WILL REPRESENT THE NUMBER OF STICKS.

	correct
(1) 183	183_______________
(2) 105	105_______________
(3) 24	24________________
	comments___________

IX (Place the following numerals before student using the plastic symbols.)

READ THIS NUMERAL.	correct	incorrect
(1) 12,306,758	______	______
(2) 1,230,675	______	______
(3) 123,067	______	______
(4) 1230	______	______
(5) 12	______	______
(6) 2	______	______

Appendix E
Place Value
Semi-Concrete

Name____________________

I	(1)	(Show photograph.) COUNT THE STICKS SHOWN IN THIS PICTURE.	counts by ones________ other________________ makes error__________
	(2)	(Show photograph.) HOW MANY BUNDLES OF STICKS ARE THERE?	five________________ doesn't know__________ hesitates____________
	(3)	IF EACH BUNDLE (point) CONTAINS TEN STICKS, HOW MANY STICKS ARE THERE?	fifty_______________ five bundles of ten___ doesn't know__________ hesitates____________
II	(1)	(Show photograph.) HOW MANY BUNDLES ARE SHOWN IN THE PICTURE?	correct______________ other________________
	(2)	IF EACH BUNDLE CONTAINS TEN STICKS, HOW MANY STICKS ARE THERE?	correct______________ other________________
	(3a)	(If correct on 2), HOW DO YOU KNOW?	10 x 12 = 120_________ add zero_____________ counts by tens________ doesn't know_________
	(3b)	(If incorrect on 2), COUNT THE STICKS BY TENS!	correct______________ pauses at 100________

III (Show photograph. Student should have paper and pencil.)

EACH OF THESE BUNDLES (point) CONTAINS TEN STICKS.

HOW MANY STICKS ARE THERE?

(Wait for oral response.)

WRITE THE NUMERAL!

forty-three__________
4 tens and 3 ones_____
other_______________
numeral_______________

IV (Show both photographs.)

EACH PICTURE (point to both) CONTAINS THE SAME NUMBER OF STICKS! THESE BUNDLES (point) CONTAIN TEN STICKS EACH. YOU MAY COUNT THE STICKS IN EITHER PICTURE.

HOW MANY STICKS ARE THERE IN EACH PICTURE? COUNT ALOUD!

counts the bundled____
by 10, 20, 30________
by 1, 2, 3___________
counts unbundled______
counts both__________
other_______________

V (Show photograph.)

(1) THERE ARE TEN STICKS (point) IN EACH OF THESE BUNDLES. HOW MANY STICKS ARE THERE?

163__________________
16 tens, 3 ones_______
other_______________

(2) WHAT WOULD YOU DO TO REMOVE 100 STICKS FROM THE PICTURE?

removes ten bundles___
other_______________

(3) HOW MANY STICKS WOULD BE LEFT IN THE PICTURE?

63___________________
counts again yes__no_

VI (Student should have paper and pencil. Show photograph.)

(1) IF THESE BUNDLES (point) EACH CONTAIN 100 STICKS AND THESE (point) CONTAIN 10 STICKS EACH, WRITE THE NUMERAL WHICH WILL SHOW HOW MANY STICKS ARE IN THE PICTURE.

forms 254____________
other________________
hesitates yes___no___

(2) (If 1 correct, show next photograph.)

DRAW A LINE AROUND THE ONE SYMBOL YOU WOULD CHANGE IN YOUR NUMERAL IF I PUT TWO MORE BUNDLES OF TENS IN THE PICTURE!

circles 4 5 2
doesn't know_________

(3) WRITE THE CORRECT NUMERAL WHICH WILL SHOW HOW MANY STICKS ARE IN THE PICTURE NOW!

writes 254___________
other________________

(4) (Show next photograph.)

DRAW A LINE AROUND THE ONE SYMBOL YOU WOULD CHANGE IN YOUR NUMERAL IF I PUT FIVE MORE STICKS INTO THE PICTURE!

circles 2 5 4
can't do it__________

(5) WRITE THE CORRECT NUMERAL WHICH WILL SHOW HOW MANY STICKS ARE IN THE PICTURE NOW!

writes 259___________
other________________
can't write it_______

(6) (Numeral is now 259. Show next photograph.)

DRAW A LINE AROUND THE ONE SYMBOL YOU WOULD CHANGE IN YOUR NUMERAL, IF I PUT 5 MORE STICKS INTO THE PICTURE!

circles 9 5 2
5 and 9______________
comment

(7) WRITE THE NEW NUMERAL.

correct______________
other________________

(8) (Show photograph.)

OUR PICTURE NOW LOOKS LIKE THIS. HOW WOULD YOU CHANGE THE PICTURE SO THAT THE PICTURE WILL SHOW YOUR NUMERAL CORRECTLY?

response

VII (Show photographs and point to each. Examiner should have paper and pencil.)

THIS PICTURE SHOWS BUNDLES OF 100 STICKS. THIS PICTURE SHOWS TENS BUNDLES. THIS PICTURE SHOWS INDIVIDUAL STICKS.

I WILL WRITE NUMERALS ON THIS PAPER AND YOU ARE TO TELL ME HOW MANY ARE NEEDED FROM EACH PICTURE TO SHOW CORRECTLY THE NUMERAL.

	correct
(1) 5	5____________________
(2) 14	14____________________
(3) 22	22____________________
(4) 206	206____________________
(5) 353	353____________________

VIII (Student should have paper and pencil. Show first photograph.)

THIS IS A PICTURE (point) OF A BUNDLE OF 100 STICKS. THESE (point) ARE TENS BUNDLES. THESE (point) ARE INDIVIDUAL STICKS.

WRITE A NUMERAL WHICH WILL REPRESENT THE NUMBER OF STICKS SHOWN IN EACH OF THE FOLLOWING PICTURES.

	correct
(1) 183	183____________________
(2) 105	105____________________
(3) 24	24____________________

IX (Show photographs. Cover all except top numeral. Each numeral is uncovered after the preceding one is read.)

READ EACH NUMERAL AS THEY ARE SHOWN TO YOU.

		correct	incorrect (write reading)
(1)	12,306,758	______	______
(2)	1,230,675	______	______
(3)	123,067	______	______
(4)	1,230	______	______
(5)	12	______	______
(6)	2	______	______

Appendix F
Place Value
Abstract

Name________________

I (1) COUNT TO TWENTY-TWO BY ONES.

correct________________

(2) WRITE A NUMERAL FOR FIVE TENS.

numeral________________

II (Show student symbol on the card.)

BILL USES ⊖ THIS SYMBOL TO REPRESENT TEN OBJECTS. HOW MANY OBJECTS ARE REPRESENTED BY TWELVE OF THESE SYMBOLS?

response________________

III (Show student the symbols.)

IF THIS ⊖ SYMBOL REPRESENTS TEN OF THESE ⋈ SYMBOLS, HOW MANY OF THESE ⋈ WOULD BE REPRESENTED BY THREE OF THESE ⋈ AND FOUR OF THESE ⊖ ?

correct________________
other_ ________________

WRITE THE NUMERAL.

numeral________________

IV (1) TWO SACKS OF PENNIES EACH CONTAIN THE SAME NUMBER OF PENNIES. ONE OF THE SACKS HAS THE PENNIES BUNDLED IN ROLLS OF FIFTY PENNIES AND THE OTHER SACK DOES NOT CONTAIN ANY BUNDLING.

HOW WOULD YOU DETERMINE HOW MANY PENNIES ARE IN EACH SACK?

count________________
other________________

(2) (If answer is "count"),

WHICH SACK WOULD YOU COUNT FIRST?

bundled________________
other________________

V (Show student the symbols.)

(1) IF EACH OF THESE SYMBOLS ⊕ REPRESENTS TEN OF ⊠ THESE SYMBOLS, HOW MANY OF THESE SYMBOLS ⊠ DO I HAVE REPRESENTED BY 16 OF THESE ⊕ AND 3 OF THESE ⊠ ?

correct_______________

other_______________

(2) WRITE THE NUMERAL.

numeral_______________

(3) I WANT TO REMOVE 100 OF THESE ⊠ . HOW MANY OF THESE ⊕ DO I NEED TO REMOVE?

correct_______________

other_______________

(4) HOW MANY OF THESE ⊠ ARE LEFT?

correct_______________

other_______________

(5) WRITE THE NUMERAL

numeral_______________

VI (1) A STORE ORDERED GOLF BALLS IN BOXES OF HUNDREDS, BOXES OF TENS, AND INDIVIDUAL BALLS. IF THEY ORDER ONE BOX OF HUNDREDS, TWO BOXES OF TENS, AND FIVE BALLS, WRITE A NUMERAL FOR THE NUMBER OF BALLS ORDERED.

(Give student scratch paper.)

correct________________
other________________

(2) THE STORE CALLED AFTER PLACING THE ORDER AND ASKED FOR TWO MORE BOXES OF TENS. CIRCLE THE ONE SYMBOL YOU WOULD CHANGE TO SIGNIFY HOW MANY THEY HAVE ORDERED NOW.

correct________________
other________________

(3) WRITE THE NEW NUMERAL.

correct________________
other________________

(4) A BOY WHO IS PACKAGING THE ORDER PUT IN SEVEN EXTRA BALLS. CIRCLE THE ONE SYMBOL YOU WOULD CHANGE TO SHOW THE NUMBER OF BALLS IN THE SHIPMENT NOW.

circles 1 4 5
4 and 5________________
reaction________________

(5) WRITE THE NEW NUMERAL.

correct________________
other________________

VII (Lay cards having the symbols before student.)

THIS SYMBOL ⊖ REPRESENTS TEN OF THESE ⊠ SYMBOLS. THIS SYMBOL ⟁ REPRESENTS ONE HUNDRED OF THESE ⊠ SYMBOLS.

TELL ME THE SIMPLEST WAY, USING NO MORE THAN NINE OF EACH SYMBOL, TO INDICATE,

(Repeat before each of the following:)

(1) 5 OF THESE ⊠

(2) 14 OF THESE ⊠

(3) 22 OF THESE ⊠

(4) 206 OF THESE ⊠

(5) 353 OF THESE ⊠

correctly illustrates

5____________________

14____________________

22____________________

206____________________

353____________________

comment________________

VIII THIS SYMBOL ⊖ REPRESENTS TEN OF THESE ⊠ SYMBOLS. THIS SYMBOL ⟁ REPRESENTS ONE HUNDRED OF THESE ⊠ SYMBOLS. I WILL PLACE CARDS IN FRONT OF YOU AND I WANT YOU TO WRITE A NUMERAL WHICH WILL REPRESENT THE NUMBER OF THESE ⊠ SYMBOLS THAT I HAVE INDICATED.

(Give student scratch paper.)

(1) One hundred eighty-three

(2) One hundred five

(3) Twenty-four

correct

____

____

____

comments________________

IX I WILL READ TO YOU A NUMBER. I WANT YOU TO WRITE A NUMERAL WHICH REPRESENTS THE NUMBER WHICH I READ.

(Give student scratch paper.) correct______

(1) TWELVE MILLION, THREE HUNDRED SIX THOUSAND, SEVEN HUNDRED FIFTY-EIGHT ____________________

(2) ONE MILLION, TWO HUNDRED THIRTY THOUSAND, SIX HUNDRED SEVENTY-FIVE ____________________

(3) ONE HUNDRED TWENTY-THREE THOUSAND, SIXTY-SEVEN ____________________

(4) ONE THOUSAND, TWO HUNDRED THIRTY ____________________

(5) TWELVE ____________________

(6) TWO ____________________

Appendix G

Behavior Observation Guide

Name____ Date______________________

Description (Appearance, facial expression, attire, posture, motility, voice quality, physical handicaps, presence of glasses, handedness, etc.):

1. Sensory and Motor Proficiency:

	Markedly Deficient	Limited	Average or Better
Vision:			
Hearing:			
Manual Control:			

2. Performance Rate:

Ext. Rapid______ Rapid______ Average______ Slow______ Ext. Slow______

3. Orientation to Examination:

______Seems to have complete understanding of nature and purpose of examination.

______Shows some insight as to purpose.

______Accepts the explanation of purpose of examination.

______Occasional evidence of distorted ideas.

______Completely misinterprets situation.

4. Initial Adjustment: Final Adjustment:

______Completely at ease, makes good social contact. ______

______Better than average social confidence. ______

______May show some anxiety, but manages to control it. ______

______Rather anxious and poorly poised. ______

______Extremely ill at ease and apprehensive. ______

5. Interest:

______Enthusiastic and absorbed.

______Definitely interested in the tests.

______Shows an adequate amount of interest.

______Lack of interest shown.

______Completely uninterested.

6. Cooperation:

______Cooperates enthusiastically -- does everything requested.

______Cooperates readily, offers no resistance.

______Generally good, but may resist certain assignments.

______Somewhat negativistic.

______Negativistic and uncooperative, reducing reliability of the test.

7. Amount of Speech:

______Moderate amount of speech, recognizing limits imposed by test routine.

______Atypical speech pattern, tending toward___volubility or___taciturnity.

______Abnormal amount of speech: ___extreme loquacity or ___extreme taciturnity.

8. Expressive Ability:

______Excellent

______Good

______Adequate

______Poor

______Very poor

9. Attention:

______So attentive to test as to be oblivious to extraneous stimuli.

______Relatively undisturbed by extraneous stimuli.

______Moderately attentive.

______Easily distracted by extraneous stimuli or inner preoccupations.

______Almost impossible to get and hold attention.

10. Self-Confidence:

______Extremely self-confident, gives replies with assurance.

______Rather self-confident and assured.

______Somewhat confident, but evinces doubts.

______Definitely inclined to distrust ability.

______Painful uncertainty and vacillation.

11. Motivation:

______Intensely motivated, but no to such a degree as to reduce efficiency.

______Strongly motivated to succeed.

______Motivated sufficiently to permit fairly reliable evaluation.

______Rather unconcerned about performance.

______Motivation completely out of proportion to the situation: ___so strong as to render patient over-anxious; ___essentially nil.

12. Effort:

______Consistently expends maximum energy to attain success.

______Works diligently on most tasks.

______Strives for success, though possibly not at full pitch.

______Works perfunctorily.

______Lackadaisical, listless, indifferent.

13. Persistence:

_______Dogged persistence; unable to give up even when failure is obvious.

_______Persists even on tasks that are too difficult.

_______Persists for a reasonable length of time.

_______Admits defeat quickly when difficulty is encountered.

_______Anticipates failure -- refuses to try.

14. Ability to Shift:

_______Extreme rigidity, perseveration of ideas.

_______Shows difficulty shifting from one idea to another.

_______Adequate amount of flexibility.

_______Shifts too readily; finds it difficult to carry one task to completio

_______Thought content unstable and fleeting; cannot hold one topic in mind.

15. Reaction to Praise and Encouragement:

_______Shows renewed or increased self-confidence, but still recognizes limits of ability.

_______Stimulated to try harder, even when approaching maximum ability level.

_______Stimulated to try a bit harder or persist slightly longer.

_______Accepts with considerable reserve; any change in motivation temporary.

_______Unmoved and unimpressed.

16. Reaction to Failure:

_______Adversely affected, manifesting emotional reactions, offering rationalizations and excuses, etc.

_______Somewhat upset by failure, tends to magnify it.

_______Shows some disappointment, but accepts it realistically as something to be expected.

_______Less disturbed than would be expected under the circumstances.

_______Unconcerned; no observable reaction.

17. Self-Criticism:

_______Markedly hypercritical.

_______Shows a general tendency to be overcritical.

_______Appraises performance accurately.

_______Shows a tendency to overrate performance.

_______Decidedly uncritical and naive.

18. Miscellaneous Indicators:

______Inappropriate laughter and private jokes.

______Irrelevant and bizarre verbalizations.

______Tengential thinking.

______Emotional outbursts.

______Hallucinations.

______Euphoria.

______Flat affect.

______Personal associations.

______Speech blocking.

19. Representativeness of Results:

______Obtained results definitely not indicative of patient's true capacity.

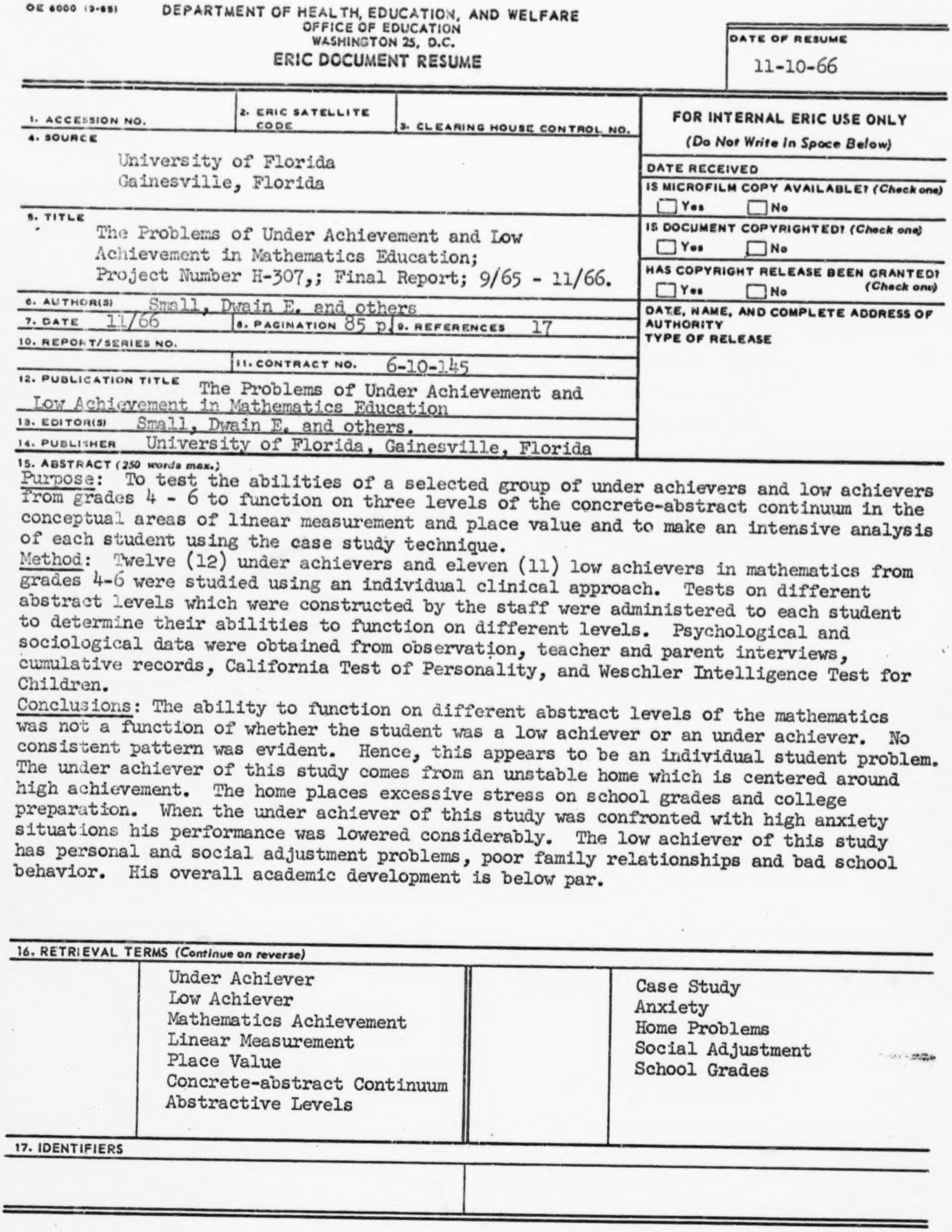
______Results on most tests contaminated and not optimum.

______Some results satisfactory, others contaminated.

______Some doubt as to complete representativeness of results.

______Obtained findings considered to be a reliable sample of patient's behavior potential.

20. Comments:



OE 6000 (3-65) **DEPARTMENT OF HEALTH, EDUCATION, AND WELFARE**
OFFICE OF EDUCATION
WASHINGTON 25, D.C.

ERIC DOCUMENT RESUME

DATE OF RESUME: 11-10-66

1. ACCESSION NO.
2. ERIC SATELLITE CODE
3. CLEARING HOUSE CONTROL NO.

4. SOURCE
University of Florida
Gainesville, Florida

5. TITLE
The Problems of Under Achievement and Low Achievement in Mathematics Education; Project Number H-307,; Final Report; 9/65 - 11/66.

6. AUTHOR(S) Small, Dwain E. and others
7. DATE 11/66
8. PAGINATION 85 p.
9. REFERENCES 17
10. REPORT/SERIES NO.
11. CONTRACT NO. 6-10-145
12. PUBLICATION TITLE The Problems of Under Achievement and Low Achievement in Mathematics Education
13. EDITOR(S) Small, Dwain E. and others.
14. PUBLISHER University of Florida, Gainesville, Florida

FOR INTERNAL ERIC USE ONLY
(Do Not Write In Space Below)

DATE RECEIVED
IS MICROFILM COPY AVAILABLE? *(Check one)* ☐ Yes ☐ No
IS DOCUMENT COPYRIGHTED? *(Check one)* ☐ Yes ☐ No
HAS COPYRIGHT RELEASE BEEN GRANTED? *(Check one)* ☐ Yes ☐ No
DATE, NAME, AND COMPLETE ADDRESS OF AUTHORITY
TYPE OF RELEASE

15. ABSTRACT *(250 words max.)*

Purpose: To test the abilities of a selected group of under achievers and low achievers from grades 4 - 6 to function on three levels of the concrete-abstract continuum in the conceptual areas of linear measurement and place value and to make an intensive analysis of each student using the case study technique.

Method: Twelve (12) under achievers and eleven (11) low achievers in mathematics from grades 4-6 were studied using an individual clinical approach. Tests on different abstract levels which were constructed by the staff were administered to each student to determine their abilities to function on different levels. Psychological and sociological data were obtained from observation, teacher and parent interviews, cumulative records, California Test of Personality, and Weschler Intelligence Test for Children.

Conclusions: The ability to function on different abstract levels of the mathematics was not a function of whether the student was a low achiever or an under achiever. No consistent pattern was evident. Hence, this appears to be an individual student problem. The under achiever of this study comes from an unstable home which is centered around high achievement. The home places excessive stress on school grades and college preparation. When the under achiever of this study was confronted with high anxiety situations his performance was lowered considerably. The low achiever of this study has personal and social adjustment problems, poor family relationships and bad school behavior. His overall academic development is below par.

16. RETRIEVAL TERMS *(Continue on reverse)*

Under Achiever
Low Achiever
Mathematics Achievement
Linear Measurement
Place Value
Concrete-abstract Continuum
Abstractive Levels

Case Study
Anxiety
Home Problems
Social Adjustment
School Grades

17. IDENTIFIERS

Figure 3. ERIC Document Resume

APPENDIX H

How I See Myself and My Family

We would like very much to know how you feel about yourself and your family. Would you please take the paper you have been given, write your name at the top, then briefly tell us how you feel about things. We would like to know how you feel about yourself, what kind of things you like to do and like to think about, and to tell us what your mother, father and brothers and sisters are like. Knowing these things will make it easier for us to understand how you learn so that we can help other pupils to learn better.